MAPPING THE PSEUDO-ARC ONTO CIRCLE-LIKE, SELF-ENTWINED CONTINUA

J. T. Rogers, Jr.

The investigation of the continuous images of the pseudo-arc has aroused much activity in this decade. Of particular interest has been the question of deciding which circle-like continua are continuous images of the pseudo-arc.

In this paper, we define the notion of a circle-like, self-entwined continuum. We find that circle-like, self-entwined continua are indecomposable, that all nonplanar, circle-like continua are self-entwined, and that the planar, circle-like, self-entwined continua separate the plane. Two of our results follow.

THEOREM. *No arc-like continuum can be mapped onto a circle-like, self-entwined continuum.*

THEOREM. *The pseudo-circle is self-entwined.*

Several known results follow as corollaries to these theorems. W. T. Ingram [6] has shown that the pseudo-arc cannot be mapped onto a nonplanar, circle-like continuum, and L. Fearnley [5] and the author [11] have shown independently that the pseudo-arc cannot be mapped onto the pseudo-circle.

In this paper, we use the methods of inverse limit spaces. The transition from chains to inverse limits is discussed in [10] and [11]. We use the terminology of [3].

A continuum is a compact, nondegenerate, connected subset of a metric space. A map is a continuous function.

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Let C denote the unit circle in the plane. Orient C so that a definite sense of rotation exists. Let C_1 and C_2 be triangulations of C, and let f: $C_1 \to C_2$ be a surjective, simplicial map.

Let v denote a vertex of C_2, and let $z_0, z_1, \cdots, z_n$ denote the vertices of C_1 that are mapped onto v, ordered by positive rotation. Suppose z_{n+1} is another name for z_0. If $f \mid [z_i, z_{i+1}]$ is surjective, call $[z_i, z_{i+1}]$ an A+ or A-, according to whether the image of $[z_i, z_{i+1}]$ emanates from v in the positive or negative direction.

If $[x_1, x_2]$ is an arc in C_1, oriented in the direction of positive rotation, we define the *degree of* $[x_1, x_2]_f$ to be the number of A+'s of $f \mid [x_1, x_2]$, diminished by the number of A-'s of $f \mid [x_1, x_2]$. Where no confusion is likely, we speak of the degree of $[x_1, x_2]$ without reference to the function f. For greater detail on these concepts, we refer the reader to [10] and [11].

The author [11] has shown that if the map f has positive degree, then there exists an integer i such that if $[z_i, z_j]$ is an arc in C_1, then the degree of $[z_i, z_j]$ is positive. The point z_i is called an *initial point of* C_1 *with respect to* f. If deg(f) = 1, then the initial point is unique [10, Lemma 6].

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