MONTGOMERY-SAMELSON COVERINGS ON SPHERES

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1. INTRODUCTION

In this note, we study maps $f: M \to N$ from a compact manifold onto a compact manifold. Such a map is called a *Montgomery-Samelson covering* if $f \mid (M - B_f)$ is a covering map onto $N - fB_f$ and $f \mid f^{-1} fB_f$ is a homeomorphism onto fB_f, where B_f is the set of points of M at which f is not a local homeomorphism. Furthermore, we assume that $\dim B_f \leq n - 2$ and that the Čech homology groups of B_f are finitely generated. For the rest of this note, f denotes such a map. All spaces, except B_f, are manifolds unless exceptions are explicitly noted. S^n denotes the n-sphere, and d $(d > 1)$ is the degree of f. We prove the following results.

THEOREM 1. *If* $f: S^n \to S^n$ *satisfies the requirements above, then* B_f *is an* $(n - 2)$*-dimensional homology sphere, modulo each prime dividing* d.

This theorem answers a question raised by H. Hopf [5, paragraph 3] and E. Hemmingsen [3, p. 328].

THEOREM 2. *If* $f: M \to S^n$ *is a Montgomery-Samelson covering and* fB_f *is a trivially knotted* p*-sphere in* S^n, *then* $p = n - 2$, *the manifold* M *is a topological sphere, and* f *is the* $(n - 1)$*-fold suspension of a* d*-to-*1 *covering map of* S^1 *on* S^1.

We adapt to the setting of codimension zero some techniques that P. L. Antonelli devised in his work on Montgomery-Samelson fiberings [1], [2]. The proof of Theorem 1 uses a special homology analogous to that of P. A. Smith [7].

2. SPECIAL HOMOLOGY

PROPOSITION. *Let* $f: M \to N$ *be a Montgomery-Samelson covering. Let* p *be some prime dividing* d. *Let* H *denote Čech homology with coefficients in* Z_p *(the integers modulo* p*). Then there exist graded* Z_p*-modules* $H^\tau(M)$, $H^\tau(M, B_f)$, $H^\sigma(M)$, *and* $H^\sigma(M, B_f)$ *such that*

(a) *for each* m, *there exist exact sequences*

$$H^\sigma_{m+1}(M, B_f) \to H^\tau_m(M, B_f) \oplus H_m(B_f) \to H_m(M)$$

and

$$H^\tau_{m+1}(M, B_f) \to H^\sigma_m(M, B_f) \oplus H_m(B_f) \to H_m(M)\,,$$

and

(b) $H^\sigma_m(M, B_f)$ *is the homomorphic image of* $H_m(N, B_f)$.

Proof. Part (a) of this theorem is proved in [4]. The homomorphism of part (b) is induced at the chain level in the simplicial case if to each simplex s in (N, B_f),

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