R-AUTOMORPHISMS OF R[[X]]

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1. INTRODUCTION

In this paper, we assume that each ring is commutative and contains an identity element. If R is such a ring, then an endomorphism ϕ of R[[X]] is said to be an R-*endomorphism* if $\phi(r) = r$ for each element r in R. The results of this paper are closely related to those of M. J. O'Malley in [1] and of O'Malley and C. Wood in [2]. Before proceeding further, we summarize some of the results of [1] and [2] that are relevant to this paper.

Let S be a ring, and suppose that S contains an element b_0 such that S is a complete Hausdorff space in the (b_0)-adic topology. In [1, Theorem 2.1], O'Malley proved that for each element $\alpha = \sum_{i=0}^{\infty} a_i X^i$ of S[[X]] with $a_0 \in (b_0)$, there exists a unique R-endomorphism ψ_α of R[[X]] such that $\psi_\alpha(X) = \alpha$; moreover, ψ_α is onto if and only if a_1 is a unit of R, and if ψ_α is onto, ψ_α is also one-to-one. Conversely, if T is a ring, and if there exists a T-endomorphism f of T[[X]] such that

$$f(X) = \alpha = \sum_{i=0}^{\infty} a_i X^i,$$

where $\bigcap_{n=1}^{\infty} (a_0^n) = (0)$, then T is complete in the (a_0)-adic topology and $f = \psi_\alpha$ [1, Theorem 4.10]. The principal question that O'Malley leaves unanswered in [1] is the following:

(*) *Suppose that* R *is a ring. If there exists an* R-*automorphism of* R[[X]] *mapping* X *onto* $\sum_{i=0}^{\infty} a_i X^i$, *does it follow that* $\bigcap_{n=1}^{\infty} (a_0^n) = (0)$?

Corollary 5.5 of [1] shows that the answer to (*) is affirmative if

$$\bigcap_{n=1}^{\infty} (a_0^n) = a_0\left[\bigcap_{n=1}^{\infty} (a_0^n)\right],$$

and hence the answer is affirmative if R is Noetherian or if a_0 is regular in R. Since a_0 must belong to the Jacobson radical of R [1, Lemma 5.1], O'Malley has characterized all R-automorphisms of R[[X]] when R is either a Noetherian ring, an integral domain, or a ring with the property that $\bigcap_{n=1}^{\infty} (a_0^n) = (0)$, for each a_0 in the Jacobson radical.

We shall refer to two results from [2]: If there exists an R-automorphism of R[[X]] mapping X onto $\sum_{i=0}^{\infty} b_i X^i$, then b_1 is a unit of R [2, Lemma 4.1]. As a consequence, it follows that if $\beta = \sum_{i=0}^{\infty} b_i X^i$ is an element of R[[X]], where b_1 is

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