AN EXTENSION OF A THEOREM OF T. ANDÔ

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In [1], T. Andô proved the following result: *Every commutative pair of contractions* $\mathcal{T} = \{T_1, T_2\}$ *in a Hilbert space* H *has a unitary dilation.*

We recall that the pair $\mathcal{U} = \{U_1, U_2\}$ is a unitary dilation (in a Hilbert space $K \supset H$) for the pair $\mathcal{T}$ if the U_j ($j = 1, 2$) are unitary operators in K, $U_1 U_2 = U_2 U_1$, and

$$P U_1^{n_1} U_2^{n_2} h = T_1^{n_1} T_2^{n_2} h \quad (n_1, n_2 \geq 0,\ h \in H),$$

where P is the orthogonal projection of K onto H.

The aim of this note is to extend this theorem.

We say that a family $\mathcal{T} = \{T_1, \cdots, T_p\}$ of linear bounded operators (in a Hilbert space H) is *cyclic commutative* if

$$\text{(c)} \qquad T_1 T_2 \cdots T_p = T_p T_1 T_2 \cdots T_{p-1} = \cdots = T_2 T_3 \cdots T_p T_1 .$$

THEOREM 1. *Let* $\mathcal{T} = \{T_1, T_2, \cdots, T_p\}$ *be a cyclic commutative family of contractions in the Hilbert space* H. *There exists a cyclic commutative family* $\mathcal{V} = \{V_1, V_2, \cdots, V_p\}$ *of isometries in a Hilbert space* $K \supset H$, *with the property that*

$$\text{(1)} \qquad P V_{i_1}^{n_1} \cdots V_{i_p}^{n_p} h = T_{i_1}^{n_1} \cdots T_{i_p}^{n_p} h \quad (n_j \geq 0,\ h \in H),$$

where $(i_1, i_2, \cdots, i_p)$ *is an arbitrary permutation of* $(1, 2, \cdots, p)$, *and where* P *is the orthogonal projection of* K *onto* H.

Proof. Let $K = \ell^2(H)$; that is, let K be the space of sequences $\{h_i\}_{i=0}^{\infty}$ ($h_i \in H$) such that $\sum_{i=0}^{\infty} \|h_i\|^2 < \infty$. For $j = 1, 2, \cdots, p$, we define $S_j \in \mathcal{L}(K)$ by the equation

$$S_j\{h_0, h_1, \cdots, h_n, \cdots\} = \{T_j h_0, 0, D_{T_j} h_0, 0, h_1, \cdots, h_n, \cdots\},$$

where $D_{T_j} = (I - T_j^* T_j)^{1/2}$.

It is obvious that the S_j are isometries in K. We consider the products

$$S_1 S_2 \cdots S_p \{h_0, h_1, \cdots\}$$
$$= \{T_1 T_2 \cdots T_p h_0, D_{T_1} T_2 \cdots T_p h_0, 0, D_{T_2} T_3 \cdots T_p h_0, 0, \cdots, D_{T_{p-1}} T_p h_0, 0,$$
$$D_{T_p} h_0, 0, h_1, h_2, \cdots\},$$

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