RESTRICTIONS OF ISOTOPIES AND CONCORDANCES

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If X and Y are polyhedra and h_0 and h_1 are piecewise-linear (PL) homeomorphisms of X onto Y, a *concordance (weak isotopy, pseudo-isotopy)* between h_0 and h_1 is a PL homeomorphism

$$H: X \times I \to Y \times I \qquad (I = [0, 1])$$

such that $H(x, 0) = (h_0(x), 0)$ and $H(x, 1) = (h_1(x), 1)$ for all $x \in X$. If, in addition, $H(x, t) = (h_t(x), t)$ for all $(x, t) \in X \times I$, then H is called an *isotopy*. If $A \subset X$, a concordance or isotopy H between h_0 and h_1 is *fixed on* A if $H(x, t) = (h_0(x), t)$ for all $(x, t) \in X \times I$.

Let M be a simply-connected PL manifold, and let m be an interior point of M. In [1], H. Gluck showed that if h_0 and h_1 are two isotopic (concordant) homeomorphisms of M, each of which leaves m fixed, then $h_0 \mid M - \{m\}$ and $h_1 \mid M - \{m\}$ are isotopic (concordant) homeomorphisms of $M - \{m\}$. We generalize as follows.

THEOREM. *Let* Q^q *be a* PL *q-dimensional manifold, and let* M^m *be a compact, connected, m-dimensional polyhedron with* $q \geq m - 3$. *Suppose that one of the following three properties is satisfied.*

a) Q *is* $(m + 1)$*-connected with* $q \geq 2m + 3$.

b) Q *is* $(m + 1)$*-connected, and* M *is a closed,* $(2m - q + 2)$*-connected* PL *manifold.*

c) Q *is* $(k + 1)$*-connected, and* M *is a* PL *manifold with* k*-spine* K^k $(k < n,\ q \geq m + k + 2)$.

Let $f: M^m \to \operatorname{int} Q^q$ *be a* PL *embedding. If* h_0 *and* h_1 *are* PL *homeomorphisms of* Q *that are the identity on* $f(M^m)$ *and are isotopic (concordant), then there exists an isotopy (concordance) between* h_0 *and* h_1 *that is fixed on* $f(M^m)$.

Remark. This theorem can be generalized to consider the case where f is an allowable embedding. We shall assume familiarity with either [3] or [7].

1. CONCORDANCES

Let $H: Q \times I \to Q \times I$ be a concordance between h_0 and h_1. Define $F: M \times I \to Q \times I$ by $F(x, t) = (f(x), t)$. To prove the theorem in this case, it suffices to find a PL homeomorphism $h: Q \times I \to Q \times I$ such that $h \mid Q \times \{0, 1\}$ is the identity map and $hHF = F$, since hH is the desired concordance.

In case a), the existence of such an h is a well-known corollary of the general-position theorem. In case b), one applies Theorem 4 of [2]. The case where M is a bounded manifold is handled by techniques from the unpublished works of J. Dancis, J. F. P. Hudson, and R. Tindell. We sketch a proof, for the sake of completeness.

Since Q is $(k + 1)$-connected, there exists a PL homotopy $f_t: M \times I \to Q \times I$ such that $f_0 = F$ and $f_1 = HF$. We may assume that there exists an $\varepsilon > 0$ such that

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