ON CONSISTENCY WITH RESPECT TO FUNCTIONALS OF ℓ - ℓ TRANSFORMATIONS

G. E. Peterson

1. INTRODUCTION

Let ℓ denote the space of absolutely convergent series of complex numbers. Many theorems concerning ℓ - ℓ methods of summation are stated relative to the *natural linear functional* $\sigma \in \ell'$, that is, the complex-valued functional defined by $\sigma(x) = \sum_n x_n$ for every $x \in \ell$. For example, H. I. Brown and V. F. Cowling have proved a theorem [1, Theorem 3] relating the concepts of perfectness and consistency (with respect to σ). One may ask what happens to these theorems if the natural linear functional is replaced by an arbitrary linear functional. The present research grew out of an attempt to answer this question as it pertains to the theorem of Brown and Cowling.

Let s denote a space of sequences, and let A and B denote matrices that represent mappings from s into ℓ; in other words, let A and B represent s - ℓ transformations. If $F \in \ell'$, we say that A and B are *F-consistent* on s provided $F(Ax) = F(Bx)$ for each $x \in s$. If A is an ℓ - ℓ transformation, we say that *F-consistency is extendable from* ℓ *to* ℓ_A (see [1] and [3] for notation not defined here) provided A and B are F-consistent on ℓ_A whenever B is an ℓ - ℓ method that is F-consistent with A on ℓ and satisfies the condition $\ell_B \supseteq \ell_A$. Brown and Cowling proved [1, Theorem 3] that σ-consistency is extendable from ℓ to ℓ_A if and only if A is perfect. We show (see Theorems 3, 4, and 5) that this result extends to a functional $F \in \ell'$ if and only if F has the representation $F(x) = \sum_n h_n x_n$, where $h \notin c_0$. We are then able to define a class of transformations that is included in the class of ℓ - ℓ transformations as a proper subset but includes the perfect transformations as a proper subset (Theorem 7). In the process of proving these theorems, we establish some other results. For example, Theorem C gives necessary and sufficient conditions for A to map the space of absolutely convergent sequences into ℓ, and Theorem 1 is a generalization of the lemma of [1].

2. MATRIX TRANSFORMATIONS OF SEQUENCE SPACES

Let E^∞ (respectively, ac, c_0, c, m) denote the space consisting of all complex sequences $x = \{x_n\}$ such that $x_n = 0$ for all but finitely many n (respectively, such that $\sum_n |x_n - x_{n+1}| < \infty$, $\lim_n x_n = 0$, $\lim_n x_n$ exists, $\sup_n |x_n| < \infty$). If s is a sequence space and $A = (a_{nk})$ is an infinite matrix, then s_A denotes the space of all sequences x such that $Ax \equiv \left\{ \sum_k a_{nk} x_k \right\} \in s$. If s_1 and s_2 are sequence spaces, we say that A defines an s_1 - s_2 transformation if and only if $Ax \in s_2$ for every $x \in s_1$. The unit sequences e^k are defined by the equations $(e^k)_n = \delta_{nk}$. The dual of the FK-space s is denoted by s'.

THEOREM A [2, p. 29]. *The matrix* A *defines an* ℓ - c *transformation if and only if*

Received July 26, 1968.