ON THE TOPOLOGY OF A DUAL SPACE

T.-S. Wu

Let G and H be locally compact, connected, topological groups. Let Hom (G, H) denote the space of all continuous homomorphisms from G into H, with the compact-open topology (in other words, convergence on compacta is uniform). We shall call Hom (G, H) the *dual space of* G *with respect to* H.

In this note, we prove that the space Hom (G, H) is locally compact provided G and H are locally compact, connected, topological groups and H is finite-dimensional. As a corollary, we obtain the result that the automorphism group A(H) is locally compact in the compact-open topology (see [3]).

The proof of our main theorem consists of two parts. First, we prove that Hom (G, H) is locally compact if both G and H are finite-dimensional. Then we prove the local compactness of Hom (G, H) in the general case.

Throughout the paper, we assume that G and H are locally compact, connected, topological groups and that H is finite-dimensional. Let R be a compact subset of G, and let V be an open subset of H. We set

$$[R, V] = \{\sigma \in \mathrm{Hom}(G, H): \sigma(R) \subseteq V\},$$

and for $\rho \in \mathrm{Hom}(G, H)$, we set

$$\langle \rho, R, V \rangle = \{\sigma \in \mathrm{Hom}(G, H): \rho(r)^{-1}\sigma(r) \in V \text{ for all } r \in R\}.$$

Then the collection

$$\{[R, V]: R \text{ is a compact subset of } G \text{ and } V \text{ is an open subset of } H\}$$

forms a basis for the topology for Hom (G, H). The collection

$$\{\langle \rho, R, V \rangle: \rho \in \mathrm{Hom}(G, H),\ R \text{ is a compact subset of } G, \text{ and } V \text{ is a neighborhood of the identity of } H\}$$

also forms a basis of the topology of Hom (G, H). Since G is connected, we know that

$$\{\langle \rho, W, V \rangle: W \text{ is a fixed compact neighborhood of the identity of } G, \text{ and } V \text{ runs through the nuclei of } H\}$$

forms a basis of the topology of Hom (G, H) [1]. If A and B are subsets of G and H, respectively, then

$$\mathrm{Hom}(G, A; H, B) = \{\sigma \in \mathrm{Hom}(G, H): \sigma(A) \subseteq B\}.$$

Received June 20, 1968.
This research was supported in part by NSF grant GP-7527.