DISTRIBUTIVE LOCAL NOETHER LATTICES

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1. INTRODUCTION

In a recent paper, R. P. Dilworth [2] introduced the concept of a Noether lattice as an abstraction of the concept of the lattice of ideals of a Noetherian ring. A Noether lattice is a modular multiplicative lattice that satisfies the ascending chain condition and in which every element is a join of elements called principal elements. The principal elements are characterized by a pair of identities that are satisfied by the principal ideals of a ring. The usual ring-theoretic definitions of the terms *local, regular, dimension,* and *rank* carry over directly to Noether lattices. In a recent paper [1], the author showed that a distributive regular local Noether lattice of dimension n is isomorphic to RL_n, the sublattice of the lattice of ideals of $F[x_1, \cdots, x_n]$ generated by the principal ideals $(x_1), \cdots, (x_n)$ under multiplication and join.

The purpose of this paper is to describe the structure of distributive local Noether lattices. Loosely this description states that each distributive local Noether lattice L is obtained from one of the lattices RL_n by identification of some of the principal elements of RL_n with an equivalence relation that preserves join, multiplication, and cancellation in nonzero products, and by the extension of this equivalence relation to all of RL_n. The equivalence classes of RL_n modulo this relation form a lattice isomorphic to L.

Section 2 of this paper contains a characterization of principal elements in a modular multiplicative lattice, which, though known, has not yet appeared in the literature.

We use the notation and terminology of [1]. In particular, E, F, H, K, and N denote principal elements. Definitions given in [1] will not be repeated here.

2. A CHARACTERIZATION OF PRINCIPAL ELEMENTS

Our first theorem shows that in the case of a modular lattice, the defining equations for principal elements can be simplified.

THEOREM 1. *Let* L *be a modular multiplicative lattice. An element* E *of* L *is principal if and only if*

$$B \wedge E = (B:E)E \quad \text{for all } B \in L \tag{2.1}$$

and

$$(BE):E = B \vee 0:E \quad \text{for all } B \in L. \tag{2.2}$$

Proof. By Corollaries 3.1 and 3.2 of [2], principal elements of L satisfy (2.1) and (2.2).

We show first that an element E satisfying (2.1) and (2.2) is join-principal, in other words, that

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