

ON THE THEORY OF SIMPLE Γ -RINGS

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1. INTRODUCTION

Let M and Γ be two additive abelian groups. If for all $x, y, z \in M$ and all $\alpha, \beta \in \Gamma$, the conditions

$$(1) \quad x\alpha y \in M,$$

$$(2) \quad (x + y)\alpha z = x\alpha z + y\alpha z, \quad x(\alpha + \beta)z = x\alpha z + x\beta z, \quad x\alpha(y + z) = x\alpha y + x\alpha z,$$

$$(3) \quad (x\alpha y)\beta z = x\alpha(y\beta z)$$

are satisfied, then, following Barnes [1], we call M a Γ -ring. If these conditions are strengthened to

$$(1') \quad x\alpha y \in M, \quad \alpha x\beta \in \Gamma,$$

$$(2') \quad \text{the same as (2),}$$

$$(3') \quad (x\alpha y)\beta z = x(\alpha y\beta)z = x\alpha(y\beta z),$$

$$(4') \quad x\alpha y = 0 \text{ for all } x, y \in M \text{ implies } \alpha = 0,$$

then M is called a Γ -ring in the sense of Nobusawa. Clearly, every associative ring A is a Γ -ring, but it need not be a Γ -ring in the sense of Nobusawa if $\Gamma = A$. In [4], Nobusawa obtained an analogue of Wedderburn's theorem, for simple Γ -rings with minimal condition on one-sided ideals. In an earlier paper, the author developed the concept of primitivity for Γ -rings, and he characterized the primitive Γ -rings in the sense of Nobusawa having minimal one-sided ideals, by means of certain Γ -rings of continuous semilinear transformations. This characterization generalized a result of Jacobson in ordinary ring theory.

In this paper, we extend the notions of simplicity and complete primeness to Γ -rings. Our definition of simple Γ -rings differs slightly from Nobusawa's original definition, and the simple Γ -rings defined by Nobusawa are now called *completely prime* Γ -rings. However, the two concepts are identical for a Γ -ring in the sense of Nobusawa with minimum condition on one-sided ideals. We study the relations among simplicity, primeness, primitivity, and complete primeness for Γ -rings. Much of the development is analogous to the corresponding part of ring theory. We also define socles for Γ -rings, and we discuss their basic properties. One of our main results is the generalized Litoff theorem for simple Γ -rings having minimal left ideals. Finally, we determine completely the one-sided ideals of a simple Γ -ring having minimal one-sided ideals.

We refer to [2] for all notions relevant to ring theory.