SEMIFREE DIFFERENTIABLE ACTIONS OF S^1 ON HOMOTOPY $(4k+3)$-SPHERES

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1. INTRODUCTION

We shall call an action of S^1 *semifree* if it is free outside the set F of fixed points. It is well-known that if S^1 acts semifreely on a homotopy $(4k+3)$-sphere Σ^{4k+3} with fixed point set F of codimension 4, then the orbit space has a natural differentiable structure and is a homotopy $(4k+2)$-sphere. In this paper, we study the semifree differentiable actions of S^1 on homotopy $(4k+3)$-spheres $(k \geq 2)$, the fixed point sets consisting of the homotopy $(4k-1)$-spheres. The only complete result in this direction is the following theorem of Montgomery and Yang [7, Theorem 3].

THEOREM. *On any homotopy 7-sphere, there are infinitely many differentiably distinct, semifree, differentiable actions of the circle group* S^1, *each having* S^3 *as the fixed point set.*

The following five theorems are immediate consequences of the main theorem.

THEOREM 1. *If there exists a semifree differentiable action of* S^1 *on a homotopy* $(4k+3)$*-sphere* Σ^{4k+3}, *and if its fixed point set is a homotopy* $(4k-1)$*-sphere* Σ^{4k-1} *and its orbit space is a homotopy* $(4k+2)$*-sphere* Σ^{4k+2}, *then there exist infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *on* Σ^{4k+3} *with fixed point set* Σ^{4k-1} *and orbit space* Σ^{4k+2}.

THEOREM 2. *Every homotopy sphere* Σ^{4k+3} *in* $b P_{4k+4}$ *admits infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *with fixed point sets of codimension* 4. *For example, let* Σ_M^{4k+3} *and* Σ_M^{4k-1} *be the Milnor spheres of dimensions* $4k+3$ *and* $4k-1$, *respectively. Then, for each integer* $n \geq 1$, *the homotopy sphere* $n\Sigma_M^{4k+3}$ *admits infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *with fixed point set* $n\Sigma_M^{4k-1}$. (For the notation $b P_n$, see [5, p. 510].)

THEOREM 3. *There exist infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *on* S^{4k+3} *with fixed point set* S^{4k-1} *and orbit space* S^{4k+2}.

THEOREM 4. (i) *For each homotopy sphere* Σ^7 *in* $4\theta_7$, *there exist infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *on* S^{11} *with fixed point set* Σ^7.

(ii) *On each homotopy sphere* Σ^{11} *in* $2\theta_{11}$, *there are infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *with orbit space* S^{10}.

(iii) *On each homotopy sphere in* $4\theta_{11}$, *there are infinitely many differentiably distinct, semifree, differentiable actions of* S^1 *with fixed point set* S^7 *and orbit space* S^{10}.

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