THE ANALYSIS OF REPRESENTATIONS INDUCED FROM A NORMAL SUBGROUP

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1. INTRODUCTION

Following the lines laid down by Clifford in [2], a number of authors have investigated the relation between representations of a group and representations of a normal subgroup. (See the standard reference [5] and the references listed there.) Recently, S. B. Conlon [3] and P. A. Tucker ([9] to [12]) have studied the decomposition of representations induced from indecomposable (and irreducible) representations of a normal subgroup, primarily through an analysis of the endomorphism ring of the induced representation in terms of the endomorphism ring of the original representation.

In the present paper we seek to enlarge and simplify these results. The relation of a group to a normal subgroup is extended somewhat to a situation involving an algebra and a subalgebra (Section 2). Associated with this algebra is a group analogous to the quotient of a group by its normal subgroup. In Section 4 we investigate a particular case in which the commuting ring of the induced module contains a crossed-product of this associated group with a division algebra in the commuting ring of the original module. The induced module turns out to be a free module over the crossed-product, and in the last three sections of the paper we use this to obtain results of Conlon and Tucker as well as some new theorems. These results give a fairly complete picture in the case where the original module is irreducible (Section 7). No restriction is made on the underlying field.

The principal results of the paper are Theorem 2, the propositions of Section 5, and Theorems 4 and 5.

In a recent paper [4], Conlon gives a more functorial approach to the relationship between submodules of an induced module and left ideals in the endomorphism ring of the induced module. (See especially Section 2.3 of [4]. There are some restrictions on the base field, and the objects of study are group rings. In some unpublished work, E. C. Dade has studied algebras axiomatized as in Section 2 of the present paper.)

The author is grateful to Professor W. F. Reynolds for his helpful advice.

2. DEFINITIONS AND PRELIMINARY RESULTS

Let K be a field, and let A be a finite-dimensional algebra over K, with an identity. In [14], K. Yamazaki introduced the concept of *ring extension*. Generalizing this idea, we make the following assumptions about A: we assume that there exists a collection of nonzero subspaces A_g of A (where the index g ranges over a finite group G with identity 1) such that

(1) $A_g A_h = A_{gh}$ $\quad (g, h \in G)$,

Received September 15, 1967.
This research was supported in part by the National Science Foundation.