THE SECTION EXTENSION THEOREM AND LOOP FIBRATIONS

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In the discussion of principal fibrations, one has to spend some time on partitions of unity in the theory of principal fibrations. Thus most of this paper originates from [2]. Because equivariant fibrations have recently attracted much interest, we have tried to use their language as a vehicle. Loop fibrations provide an easy application of this theory, and we can generalize and correct a result of [1].

Let H_0 be a strictly associative H-space with strict unit element ε. Except where we state the contrary, all discussions in this paper are restricted to the category $\mathfrak{C}_0$ of topological spaces, with an operation of H_0 and with continuous maps that are compatible with the operations involved. All operations are assumed to be associative, with ε acting as the identity map.

If X is an object in $\mathfrak{C}_0$, the action of H_0 on X is in general not a morphism in $\mathfrak{C}_0$; in particular, the multiplication of H_0 is an action on H_0 but not a morphism in $\mathfrak{C}_0$. When we refer to H_0 as object in $\mathfrak{C}_0$, we refer to this action on H_0.

The unit interval I in this category is [0, 1], together with the trivial operation $h(x) = x$ for all $h \in H_0$ and $x \in [0, 1]$. If X and Y are in $\mathfrak{C}_0$, the product of X and Y is $X \times Y$, together with the diagonal action. Thus we can use haloing functions, halos [2, Definition 2.1], and homotopies in $\mathfrak{C}_0$ (haloing functions are obviously constant on orbits).

Following a suggestion of D. Puppe, we consider the following reformulation of the section extension property.

Let $E \xrightarrow{p} B$ be a map onto B in $\mathfrak{C}_0$; then a cross-section $s: B \to E$ is a map such that the diagram

$$\begin{array}{ccc} B & \xrightarrow{s} & E \\ & {\scriptstyle 1_B}\searrow \quad \swarrow{\scriptstyle p} & \\ & B & \end{array}$$

is commutative. If $E' \xrightarrow{p'} B'$ is also a map onto B' in $\mathfrak{C}_0$, we consider the two diagrams

$$(1)\qquad \begin{array}{ccc} E & \overset{\sigma}{\dashrightarrow} & E' \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p'} \\ B & \xrightarrow{f} & B' \end{array}, \qquad\qquad (2)\qquad \begin{array}{ccc} E & \overset{\sigma}{\dashrightarrow} & E' \\ \downarrow{\scriptstyle p} & & \downarrow{\scriptstyle p'} \\ B & \overset{\bar{\sigma}}{\dashrightarrow} & B' \end{array}$$

We have two problems:

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