PARTS IN ANALYTIC POLYHEDRA

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1. INTRODUCTION

The nontrivial Gleason parts in the maximal ideal spaces of several large classes of function algebras have been shown to carry analytic structures. For example, Wermer [7] has shown that a nontrivial part of a Dirichlet algebra is the continuous one-to-one image of the open unit disc in the complex plane, all functions in the algebra being analytic on this disc in the obvious sense. Hoffman [6] obtained the same result for logmodular algebras. Although the general conjecture that a nontrivial part always carries analytic structure does not hold (see [2]), the class of algebras for which it is true is much larger than the cases (of essentially one complex dimension) mentioned above.

In this paper we examine the parts of algebras on analytic polyhedra in n-dimensional complex space $\mathfrak{C}^n$. In particular, we consider the function algebra that is the uniform closure on an analytic polyhedron of the functions holomorphic in a neighborhood of the polyhedron. As one would hope, the parts carry analytic structures on which the functions in the algebra are analytic. We show that the nontrivial parts are analytic subvarieties in relatively compact open sets in $\mathfrak{C}^n$. Moreover, any part containing an isolated point (that is, a point at which the subvariety describing the part is 0-dimensional) must reduce to the trivial part consisting of just the point itself. Actually, by appealing to Hoffman's characterization (in terms of analytic varieties) of points in the minimal boundary of analytic polyhedra [5], we can make a stronger statement: *an isolated point of a part must lie in the minimal boundary.* Viewed as a statement about the connectedness of parts, this says that a component of a nontrivial part cannot consist of a point. We would like to show that in general the parts of polyhedra are connected analytic varieties, but we have not been able to do this.

In Section 2, we include a brief review of the relevant definitions and terminology as well as a summary of the properties of analytic subvarieties that we use later. The main tool, which shows that connected analytic subvarieties are always contained in a single part with respect to the algebra of bounded holomorphic functions, is established in Section 3. The final section contains the applications to analytic polyhedra.

2. PARTS AND ANALYTIC SUBVARIETIES

If X is a compact Hausdorff space and A is a function algebra on X, then there exists a natural equivalence relation on the space M_A of maximal ideals of A. To define this equivalence relation, we identify the set of nonzero multiplicative linear

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