A CATEGORY SLIGHTLY LARGER THAN THE METRIC AND CW-CATEGORIES

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1. INTRODUCTION

A number of attempts have been made recently to construct a category suitable for algebraic topology. The class M_0 of metric spaces, despite its nice topological properties, is unsuitable because mapping cylinders cannot in general be formed in M_0, and M_0 does not contain all the CW-complexes. Both of these difficulties stem from the fact that M_0 is not closed under adjunction and weak union. In this paper we study the smallest category M that contains M_0 and is closed under adjunction and weak union. M-spaces can be constructed from metric spaces by a process analogous to the way CW-complexes are built up from cells. Many of the convenient separation properties of metric spaces, such as paracompactness, are shared by M-spaces.

Our work is related to that of Borges [1], Michael [10], and Steenrod [12]. In fact, M is a subcategory of Steenrod's CG-category; finite products and subspaces in M are exactly those of CG. In addition, every M-space is a stratifiable space of Borges, and every separable M-space is an $\aleph_0$-space of Michael.

One of the main reasons for introducing the category M is that it provides a natural setting for Hanner's generalization of Whitehead's extension of a theorem due to Borsuk. Hanner's result states roughly that a space obtained by adjoining an ANR(M_0) to an ANR(M_0) along an ANR(M_0) is itself an ANR(M_0), provided that it is metrizable. In M, this result holds without qualifications: a space obtained by adjoining an ANR(M) to an ANR(M) along an ANR(M) is itself an ANR(M). This is the main result of the last section of the paper.

After stating some preliminary definitions and results in Section 2, we define the category M in Section 3 and show that M is closed under adjunction and weak union in Section 4. In Section 5, we discuss the category CG of compactly generated spaces (k-spaces) and show that M is a subcategory of CG. We consider subspaces, product spaces, and function spaces in Sections 6 to 8. Section 9 deals with separable M-spaces and their relation to $\aleph_0$-spaces. We obtain some basic results in the theory of retracts in M in Section 10.

2. PRELIMINARIES

By a *space* we shall mean a topological space. A *pair* (Y, B) is a space Y together with a closed subset B. If (X, A) and (Y, B) are pairs such that $X \subset Y$ and $A = X \cap B$, then (X, A) is called a *subpair* of (Y, B). (Our definition of "subpair" is more restrictive than the usual definition, which requires only that $X \subset Y$ and $A \subset B$.) If, in addition, X is closed in Y, then (X, A) is a *closed subpair* of (Y, B). A *map* is a continuous function. All neighborhoods are open. We denote the interval [0, 1] by I.

Received September 14, 1967.

This research was partially supported by the National Science Foundation Grant GP-6040.