SOME n-DIMENSIONAL MANIFOLDS THAT HAVE THE SAME FUNDAMENTAL GROUP

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The formula

$$\begin{aligned} x_1 &\to x_1 \cos\theta + x_2 \sin\theta, \\ x_2 &\to -x_1 \sin\theta + x_2 \cos\theta, \\ x_3 &\to x_3, \\ &\cdots \\ x_n &\to x_n \end{aligned}$$

defines a rotation of n-dimensional euclidean space S about the (n - 2)-dimensional subspace $A = \{(x_1, \cdots, x_n) \mid x_1 = x_2 = 0\}$, which we shall denote by spin_θ. It maps the (n - 1)-dimensional half-space

$$H_\theta = \{(x_1, \cdots, x_n) \mid x_1 = \rho\cos\theta,\ x_2 = \rho\sin\theta,\ \rho \geq 0\}$$

onto the (n - 1)-dimensional half-space $H_0 = \{(x_1, \cdots, x_n) \mid x_1 \geq 0,\ x_2 = 0\}$. The point at infinity is supposed to be included, so that S and A are spheres and each H_θ is a cell whose boundary ∂H_θ is A. An (n - 2)-dimensional sphere L in the finite part of S will be called a *deform-spun sphere* if $L \cap A$ is an (n - 4)-dimensional sphere and if for each θ the intersection of L and H_θ is an (n - 3)-dimensional cell bounded by $L \cap A$. The deformation referred to is the closed isotopical deformation $K_\theta = \mathrm{spin}_\theta\, L \cap H_\theta$ $(0 \leq \theta \leq 2\pi)$ of K_0 in H_0. (During this deformation, the boundary $\partial K_0 = L \cap A$ remains fixed.) The *spun sphere* defined by Artin [1] in 1925 is, of course, the deform-spun sphere whose deformation is the stationary deformation $K_\theta = K_0$. If the deformation K_θ is stationary outside some (n - 1)-dimensional cell C whose boundary ∂C intersects K_0 at diametrically opposite points p, q of ∂C and may be described topologically inside C as the rotation of C about its axis $\overline{pq}$ through the angle $q\theta$, then the deform-spun sphere $L = L_q$ is called a q-*twist-spun sphere*. (The rotation of S is the *spin*, and the rotation of C is the *twist*.) In another paper [3], I have shown that there exist deform-spun spheres that are not twist-spun spheres.

The ν-fold cyclic covering of S branched over L_q is a closed orientable n-dimensional manifold $\Sigma = \Sigma_{\nu,q}(K_0)$. The part of Σ that lies over L is an (n - 2)-dimensional sphere Λ.

THEOREM. *The fundamental group* $\pi(\Sigma_{\nu,q})$ *of* $\Sigma_{\nu,q}$ *depends (for given* K_0*) only on the greatest common divisor* d *of* ν *and* q. *In particular,* $\Sigma_{\nu,q}$ *is simply connected whenever* d = 1.

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