MAXIMAL LINEAR MAPPINGS AND SMOOTH SELECTION OF MEASURES ON CHOQUET BOUNDARIES

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1. This paper answers a question raised by L. Bungart and H. Bauer [2, p. 155] concerning the algebra A(D) of functions that are holomorphic in a bounded open set $D \subseteq C^n$ and continuous in $\overline{D}$. Roughly, the question is *whether we can select positive, mutually absolutely continuous representing measures for the points of* D *in a smooth manner, and in a way that keeps the measures concentrated on the Choquet (minimal) boundary of* $\overline{D}$. Bungart showed that such a selection is possible if we replace "Choquet boundary" by "Šilov boundary" and interpret "smooth" to mean (norm-) harmonic. In his dissertation [6] (see also [5]), Hinrichsen showed that we can even replace "smooth" by "holomorphic," provided we make the obviously necessary change from "positive" to "complex," and alternatively, that we can replace "smooth" by "pluriharmonic" and merely change "positive" to "real."

We shall show here that the original question has an affirmative answer if we interpret "smooth" to mean "harmonic," and that with this interpretation of smoothness, we can replace "representing measures" with "Jensen measures" provided we thicken the Choquet boundary sufficiently to make it capable of supporting Jensen measures at all [4]. Most of the theorems below belong to the study of vector lattices; the argument that selects the measures as harmonic functions of the points they represent pivots on two facts: the bounded harmonic functions on D form an order-complete vector lattice, and the maximality techniques that produce scalar measures concentrated on distinguished boundaries can be adapted to linear mappings taking their values in order-complete vector lattices. Proofs that resemble closely the arguments used in working with scalar measures will be sketched rather than given in detail.

We follow [10] in terminology and notation for vector spaces (especially ordered vector spaces), and we shall often use [10] in place of primary references. Similarly, we shall follow [1] for integration theory, and [7] for the approach to the Choquet boundary that has become standard. However, our Choquet-boundary arguments will follow those of [4], since these are more general and can be used in the construction of Jensen measures. We shall need three further notational conventions: if E and F are ordered (real) vector spaces and $T: E \to F$ is a linear mapping, T will be called *order-bounded* if it is a difference of positive linear mappings. If L is an order-complete vector lattice, then the subspace of L^* consisting of all $F \in L^*$ for which

$$\langle \sup \mathfrak{F}, F \rangle = \lim_{f \in \mathfrak{F}} \langle f, F \rangle$$

whenever $\mathfrak{F}$ is an upward-directed majorized subset of L will be called the *order subdual* of L and denoted by L_*; the cone of nonnegative elements of L_* will be called L_*^+. (We have chosen these definitions in order to eliminate unnecessary discussions of existence and uniqueness of positive decompositions.) Finally, if a

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