DIMENSIONS OF COMPACT TRANSFORMATION GROUPS

L. N. Mann

1. INTRODUCTION

A well-known result of Montgomery and Zippin [6], [7] states that the dimension of a compact Lie group of homeomorphisms acting effectively on a connected n-dimensional manifold cannot exceed $n(n+1)/2$. This result does not hold for actions on nonmanifolds. It is easy to construct effective actions of tori of arbitrarily high dimension on finite connected 2-complexes, and in fact even an effective action of the infinite-dimensional torus on a compact connected 2-dimensional space. For example, consider the 2-complex consisting of a line and r disjoint closed discs with centers on the line. We obtain an effective action of the r-torus T^r on this space by leaving the line point-wise fixed and letting the i^{th} factor group of T^r act as the group of rotations on the i^{th} disc while leaving all other discs pointwise fixed. Clearly, this action has r distinct isotropy subgroups (excluding T^r itself). In Section 2 we consider actions of a compact transformation group G on a space X, and we investigate the connection between the dimension of G and the isotropy structure of the action. The results are simple to state, and the proofs are straightforward.

In [4] it was shown that for an effective action of a compact Lie group H on a connected n-manifold, many dimensions of H less than $n(n+1)/2$ are also excluded. In Section 3 we show that the same pattern of gaps in dimension occurs for transitive actions of compact non-Lie groups.

Finally, in Section 4 we investigate actions of compact connected non-Lie groups on manifolds. It is of course an unsettled question whether compact non-Lie groups can act effectively on manifolds. By a result of Bredon [2], a compact connected non-Lie group acting on an n-manifold has orbits of dimension at most $n - 3$. Using Bredon's results and the results of Section 3, we show that a compact non-Lie group acting effectively on a connected n-manifold has dimension at most $(n-4)(n-3)/2 + 1$.

We assume that all transformation groups are metrizable and that all spaces are separable and metrizable. The reader is referred to [6] or [7] for terms such as *effective*, *free*, *transitive*, *orbit*, *isotropy* or *stability subgroup*, and *orbit space*. The author is grateful to Professor T. S. Wu for his help and encouragement, and in particular for his clever suggestions in the proof of Lemma 1.

2. DIMENSION AND ISOTROPY STRUCTURE

It is known that a compact group G acting transitively and effectively on a finite-dimensional space X is finite-dimensional [6, Theorem 4], [7, p. 239]. In fact, even more is true: $\dim G \leq n(n+1)/2$, where $n = \dim X$ [6, Theorem 10], [7, p. 243]. However, since there appears to be a slight inaccuracy connected with the proof of Theorem 10 in [6], we prefer to give an alternate proof based on the following theorem. A transformation group G on a space X is said to be *almost effective* if

Received December 10, 1966.
This research was supported in part by NSF grants GP-4125 and GP-5972.