INVERSES OF EUCLIDEAN BUNDLES

J. M. Kister

To Raymond L. Wilder on his seventieth birthday.

For each Euclidean bundle or microbundle it is useful to find another bundle of the same type, called an *inverse bundle*, such that the Whitney sum of the two is a trivial bundle. Milnor in [4] ingeniously showed how to construct an inverse to a microbundle over a finite-dimensional, locally finite, simplicial complex. Here we give a short and elementary proof of the existence of inverses for Euclidean bundles over paracompact spaces having a finiteness condition. This contains Milnor's result, since one may regard a microbundle as a Euclidean bundle [2]. Hirsch [1] has also developed a new proof of the existence of the inverse of a bundle over a polyhedron, in his work on the stable existence and stable isotopy of normal microbundles.

Terminology. By *Euclidean bundle* we mean a fibre bundle (in the sense of Steenrod [5]) whose fibre is Euclidean space R^n and whose structural group is $H_0(R^n)$, the group of all homeomorphisms of R^n leaving the origin fixed, and provided with the compact-open topology. Other bundle terminology will also be taken from [5]. For microbundle terminology, see [4]. The identity map on a space will be denoted by id, the unit interval by I.

Define maps c and p of $H_0(R^n) \times H_0(R^n)$ into $H_0(R^{2n})$ by

$$c(f, g) = (g \circ f) \times \mathrm{id}: R^n \times R^n \to R^n \times R^n \quad \text{and} \quad p(f, g) = f \times g.$$

LEMMA 1. p *is homotopic to* c.

Proof. Let θ_t (t in I) be in SO(2n), and suppose that $\theta_0 = \mathrm{id}$ and $\theta_1(x, y) = (-y, x)$ (x, y in R^n). Define $\phi_t: H_0(R^n) \times H_0(R^n) \to H_0(R^{2n})$ by

$$\phi_t(f, g) = \theta_t^{-1} \circ (\mathrm{id} \times g) \circ \theta_t \circ (f \times \mathrm{id}).$$

Then ϕ_t (t in I) is the desired homotopy with $\phi_0 = p$ and $\phi_1 = c$.

Remark 1. If the homomorphism p is restricted to $G \times G$, where G is a subgroup of $H_0(R^n)$, and if K is a subgroup of $H_0(R^{2n})$ containing both SO(2n) and $p(G \times G)$, then the homotopy constructed above assumes values in K. Examples of this occur when G and K are the orthogonal, rotation, or stable homeomorphism groups in dimensions n and 2n, respectively.

LEMMA 2. *Let* ξ^k *and* η^ℓ *be two Euclidean bundles (of dimension* k *and* ℓ, *respectively) over a space* B. *Suppose that* B *is the union of two open sets* U *and* V, *and that* ξ *and* η *are both trivial over* U *and* V. *Let the coordinate transformations for* ξ *and* η *be given by* $f: U \cap V \to H_0(R^k)$ *and* $g: U \cap V \to H_0(R^\ell)$, *respectively. Then the Whitney sum* $\xi \oplus \eta$ *is also trivial over* U *and* V, *and the coordinate transformation may be taken to be* $h: U \cap V \to H_0(R^{k+\ell})$, *where* $h(b) = f(b) \times g(b)$.

The proof is straightforward, and we omit it.

Received August 9, 1966.
This research was partially supported by NSF Grant GP4151.