CRITERIA FOR A 2-SPHERE IN S^3 TO BE TAME MODULO TWO POINTS

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Dedicated to Professor Raymond L. Wilder on his seventieth birthday.

1. INTRODUCTION

Examples have been described by Fox and Artin [12] of wild 2-spheres in a 3-sphere S^3 that are locally tame except at a finite number of points. Harrold and Moise [14] have shown that at each of its points such a 2-sphere S must be locally tame from at least one of its complementary domains. Furthermore, if S has at most one point where it is wild from the component U of $S^3 - S$, then U is an open 3-cell [10, Theorem 1], [19, Corollary 2.4]. Sikkema [20] has studied a duality between spheres and arcs in E^3 that are locally tame except at one point. In this paper, we present some conditions which imply that a 2-sphere in S^3 has at most two wild points.

In Section 4, we use Theorem 1 to investigate the following question raised by Bing in [6]: Is a 2-sphere S in S^3 tame if it can be pierced along each arc in it by a tame disk? We do not answer this question, but we obtain an affirmative answer (Theorem 5) with the additional hypothesis that each component of $S^3 - S$ be an open 3-cell.

Hempel [15] has raised the following question: Is a 2-sphere S tame in S^3 if, for each $\varepsilon > 0$ and each component U of $S^3 - S$, there exists a map of S into U that moves no point more than a distance ε? He recently obtained an affirmative answer under an additional hypothesis [16]. In Section 4 we impose the alternative additional hypothesis that S can be pierced with a disk on each tame arc on S, and we observe (Theorem 6 and Corollary 3) that with this alternative additional hypothesis an affirmative answer follows from a combination of the Sphere Theorem [18], Theorem 4, and one of Hempel's recent results [16, Corollary 2].

In Section 5, we indicate how some of these results for a 2-sphere in S^3 can be adapted to a 2-manifold in a 3-manifold.

2. DEFINITIONS AND NOTATION

Let S be a 2-sphere in S^3, and let U be a component of $S^3 - S$. We define S to be *locally tame from* U *at the point* $p \in S$ if there exist a 3-cell K and a disk D such that

$$K \cap S = D, \quad p \in \operatorname{Int} D, \quad D \subset \operatorname{Bd} K, \quad K - D \subset U.$$

The sphere S is defined to be *tame from* U if S is locally tame from U at each point of S. This is equivalent to requiring that $S \cup U$ be a 3-manifold with boundary, and to requiring that $S \cup U$ be a 3-cell [1], [8], [17]. Furthermore, S is *tame* if it

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