THE FUNCTOR [, Y] AND LOOP FIBRATIONS, I.

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Dedicated to R. L. Wilder on his seventieth birthday.

1. INTRODUCTION

There are several ways to define loop fibrations and to compare them with principal fibre bundles. Definition 1 of this paper (see Section 3) is motivated by the kind of classification theorem we obtain: If $\Omega(Y, y_0)$ is the space of loops in Y based at y_0, and X is an arbitrary topological space, then the equivalence classes (Definition 3) of our loop fibrations are in one-to-one correspondence with the homotopy classes of maps from X to Y.

The maps that we admit between loop fibrations are analogous to principal maps: A principal map restricted to a fiber of a principal bundle is essentially given by a "left translation" by an element of the group. This leads to Dold's notion of a functional bundle [1, p. 249, proof of 7.5]. The same idea can be used to define functional fibrations for loop fibrations. It is interesting that in both cases there is a "universal" function space of fiber maps that is of the same homotopy type as the "classifying" space.

For the loop fibrations, the "universal" function space is well known: it is the space of all paths in the classifying space.

2. NOTATION AND BASIC CONCEPTS

Let Y be a pathwise connected topological space. A *path* W in Y is a pair (w, r) consisting of a continuous map $w: R^+ \to Y$ (R^+ is the space of nonnegative real numbers) and a number r in R^+ such that $w(t) = w(r)$ whenever $t \geq r$. The space of paths in Y is defined by

$$MY = \{W \mid W \text{ is a path in } Y\}.$$

Its topology is the subspace topology of $Y^{R^+} \times R^+$, Y^{R^+} having the compact-open topology.

Let $W_1 = (w_1, r_1)$ and $W_2 = (w_2, r_2)$ be paths in Y such that $w_1(r_1) = w_2(0)$. We define the sum $\mu(W_1, W_2) = W_1 + W_2 = (w_1 + w_2, r_1 + r_2)$ of the two paths by the formula

$$(w_1 + w_2)(t) = \begin{cases} w_1(t) & (0 \leq t \leq r_1), \\ w_2(t - r_1) & (r_1 \leq t). \end{cases}$$

The addition is not commutative, but it is continuous and associative whenever defined.

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