THE REGULAR CONVERGENCE THEOREM

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Dedicated to R. L. Wilder on his seventieth birthday.

INTRODUCTION

The concept of regular convergence dates back to Whyburn [15], Wilder [14], and White [13]. A powerful theorem of E. E. Floyd [6, (2.3)], relating the Čech homology groups of compact Hausdorff spaces and their finite closed coverings, unifies several other theorems on regular convergence. The cohomology version of this theorem by Floyd was formulated and proved by E. Dyer [4, Theorem 1] by means of Leray's sheaf theory. This was the key theorem upon which Floyd based his proof of the major part of the conjecture by D. Montgomery that any compact Lie group acting on a compact manifold has only a finite number of conjugate classes of isotropy subgroups [6, Chapters 4 and 5].

The purpose of the present paper is to extend the theorem of Floyd to a larger class of spaces (see Theorems 3.3 and 3.6 below). As an application, we prove that the Borel-Moore homology groups with compact supports are naturally isomorphic with the singular homology groups on the class of locally compact Hausdorff HLC spaces (see Theorem 4.2).

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1. AXIOMATIC HOMOLOGY THEORY

We shall generally follow the notation and terminology of Eilenberg and Steenrod in [5]. Thus, a homology H_* is defined on an admissible category $\mathscr{A}$ so that it satisfies the seven axioms. Typical categories $\mathscr{A}$ with which we shall be concerned are $\mathscr{A}_H$, which consists of all Hausdorff pairs and all continuous maps between such pairs, and $\mathscr{W}$, which consists of pairs (X, A) such that X and A are of the homotopy type of a CW-complex and all continuous maps between such pairs. We adopt the convention that whenever we apply a functor to an object, the object is assumed to be in the category on which the functor is defined.

We shall consider some additional axioms for homology theory H_*.

Compact Support Axiom. If $z \in H_q(X, A)$, there exists a compact pair $(X', A') \subset (X, A)$ such that $z \in \mathrm{Im}(H_q(X', A') \to H_q(X, A))$.

A homology theory satisfying this axiom is called a *homology theory with compact supports*. Clearly, the singular homology theory over any coefficient group satisfies this axiom. Later in this section we shall consider another homology theory with compact support, namely the one considered by Borel and Moore in [3].

In [10], J. Milnor considered the following axiom for homology theory H_*.

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