LOOSELY CLOSED SETS AND PARTIALLY CONTINUOUS FUNCTIONS

Gordon T. Whyburn

Dedicated to R. L. Wilder on his seventieth birthday.

1. INTRODUCTION

In this paper we study certain pseudo-closedness properties of sets, similar to semi-closedness [11], in relation to partial continuity of functions. We characterize peripheral continuity of multifunctions and functions in terms of countervariance of such properties. The monotone-light factorization theorem [1], [2], [9], [10], [12] is presented in a new and improved form; in this form, it includes in one simple statement not only all previous versions for continuous functions known to the author, but also Hagan's recent extensions to peripherally continuous and connectivity functions [3]. By using cohesion properties closely related to unicoherence, we display relations between these two types of functions in a new setting. Our new theorems include as special cases recent results of Hagan [4], Long [7], and the author [15]. These are extended to multifunctions, and they assert approximately that peripherally continuous functions are connectivity functions. Also, we give a new and simplified proof for the reverse implication as established by Hamilton [6] and Stallings [8].

By X, Y, $\cdots$ we denote topological spaces with the usual open-set topology, additional restrictions being clearly stated when they are imposed. Functions $f: X \to Y$ are always single-valued, except when they are specifically called multifunctions. The double arrow $f: X \Rightarrow Y$ means that the relation is from X *onto* Y. A *mapping* is always a *continuous* function. A function $f: X \to Y$ is *monotone* provided each point-inverse $f^{-1}(y)$ is a continuum, that is, a compact connected set; and f is *light* provided each $f^{-1}(y)$ $(y \in Y)$ is totally disconnected.

A set M is *totally separated* provided there exists a separation of M between any two of its points, or, equivalently, provided each point of M is a quasi-component of M. A space X is *completely normal* provided any two separated sets in X lie in disjoint open sets in X.

A connected set is *cyclic* provided it has no cut-point. A *region* in a space X is a connected open set in X. The *boundary* of an open set U will be denoted by Fr(U).

2. LOOSELY CLOSED AND RELATED SETS

A set S in a topological space X is semi-closed (see [11, p. 131]) provided each of its components is closed and each convergent sequence of its components whose limit set intersects X - S converges to a single point of X - S. We shall define and use two related but stronger properties.

A point p is called an *adhesion point* of a set M provided there exists a point q in $\overline{M}$ - p that is not separated from p in M + p + q. A set is *loosely closed*

Received July 11, 1966.