ε-MAPPINGS AND GENERALIZED MANIFOLDS

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Dedicated to R. L. Wilder on his seventieth birthday.

INTRODUCTION AND RESULTS

The purpose of this paper is to show that n-dimensional absolute neighborhood retracts that admit ε-maps onto closed orientable n-manifolds, for arbitrarily small $\varepsilon > 0$, are necessarily orientable generalized n-manifolds in the sense of Wilder. In a sequel to this paper we shall show that if one omits the orientability hypothesis, then one obtains locally orientable generalized n-manifolds.

All spaces considered are subsets of compact metric spaces. A map $f: X \to Y$ of a space X onto Y is an *ε-map* ($\varepsilon > 0$) provided diam $f^{-1}(y) < \varepsilon$, for each $y \in Y$. If Π is a class of compact polyhedra, we say that X is *Π-like* provided for each $\varepsilon > 0$ there exist a polyhedron $P \in \Pi$ and an ε-mapping $f: X \to P$ onto P (P and f depend on ε) (see [14, Definition 1]). By a *(closed)* n-*manifold* we mean a (compact) triangulable manifold without boundary having covering dimension n. We are interested here in Π-like continua, where $\Pi = \mathfrak{M}^n$ is the class of all closed, connected, orientable n-manifolds.

Homology and cohomology modules H_r and H^r are taken in the sense of Čech, based on arbitrary open coverings as in [8]. Given a principal ideal domain L, we say that a compact space X is *homology locally connected up through dimension* n *over* L (written lc_n^L) provided for each $x \in X$ and each open set $U \subset X$ about x there exists an open set V about x ($V \subset U$) such that

$$i_r^{VU} = 0 \qquad (0 \leq r \leq n),$$

where

$$i_r^{VU}: H_r(V; L) \to H_r(U; L)$$

is the homomorphism induced by inclusion $i_{VU}: V \to U$. In dimension zero we use augmented homology. It is well known that every locally contractible space X, and *a fortiori* every ANR, is lc_n^L for each L.

For open sets U of a compact space X we consider cohomology modules with compact supports

$$H_c^r(U; L) = H^r(X, X \setminus U; L)$$

(see for example [20, p. 248]). For open sets U and V ($V \subset U$), the inclusion map $i_{VU}: V \to U$ induces homomorphisms $i_{VU}^r: H_c^r(V; L) \to H_c^r(U; L)$.

The rth local co-Betti number at $x \in X$ over L, denoted by $p^r(x, X; L)$, is defined as follows: $p^r(x, X; L) = k$ means that for each open set U about x there exist

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