MAXIMAL IDEALS IN THE ALGEBRA OF BOUNDED HOLOMORPHIC FUNCTIONS

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1. INTRODUCTION

Let B(X) be the ring of bounded holomorphic functions on an open Riemann surface X, and assume that B(X) is a proper extension of the complex numbers $\mathbb{C}$. With respect to the supremum norm, B(X) is a Banach algebra. (Because B(X) is semisimple, this is the only norm, up to equivalence.) Let $\mathscr{M}(X)$ be the maximal ideal space of B(X), endowed, as usual, with the weak-star topology. For $x \in X$, define

$$M(x) = \{f \in B(X) \mid f(x) = 0\}.$$

M(x) is a maximal ideal of B(X). We call such maximal ideals of *type* I; all others are called of *type* II.

In this paper, we obtain several characterizations of the ideals of type I. We must assume, however, that X is a relatively compact domain of another surface W, and that either the boundary of X in W consists of analytic simple closed curves or every boundary point is an essential singularity of some bounded holomorphic function on X.

2. PRELIMINARIES

Definition 1. Let W be any Riemann surface. By a *bounded* domain of W we mean a domain $X \subset W$ for which Cl X is compact and $\mathrm{Cl}\, X \neq W$ (Cl X denotes the closure of X in W). A domain X is called a *finite* domain if it is bounded and the boundary of X in W equals the boundary of W - X and consists of a finite number of analytic simple closed curves.

If X is a domain of a compact Riemann surface W, and if the complement of X in W consists of a finite number of simply connected, nondegenerate continua, then it follows from the uniformization theory for Riemann surfaces that X is conformally equivalent to a finite Riemann surface. Our results for finite surfaces remain valid for this kind of surface.

LEMMA 1. *Let* X *be a bounded domain of a Riemann surface* W, *and let* $x \in \mathrm{Cl}\, X$. *Then there exists an* $f \in B(X)$ *such that*

(1) f *is analytic in a neighborhood of* Cl X,

(2) f *has a simple zero at* x *and no other zeros, and*

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