ON EXTENSIONS OF LATTICES

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Let k be an algebraic number field of finite degree, o a Dedekind-ring with quotient field k, Γ/k a finite-dimensional semi-simple algebra over k, and R an o-order in Γ. We consider R-lattices M, N, that is, finitely generated unitary R-modules that are torsion-free as o-modules. D. G. Higman has constructed an ideal $i(R) \neq 0$ in o such that $i(R)\,\mathrm{Ext}_R^1(M, N) = 0$ for all R-lattices M and N (see Curtis and Reiner [1, p. 522]). In particular, if G is a group of order n and R = oG, then i(R) = (n). A refinement of this has been established by Reiner [3]: If kM or kN affords an absolutely irreducible representation of G of degree m, then

$$\frac{n}{m}\,\mathrm{Ext}_R^1(M, N) = 0.$$

In this note, by embedding R in a maximal order $\mathfrak{O}$, we construct an ideal F(R) in the center of R that annihilates $\mathrm{Ext}_R^1(M, N)$ for arbitrary R-lattices M and N. The corresponding o-ideal $f(R) = F(R) \cap o$ may be a proper divisor of i(R) and may even contain fewer prime ideals. An even better annihilator of $\mathrm{Ext}_R^1(M, N)$ may be constructed if kM or kN does not afford a faithful representation of Γ, that is, if eM = M or eN = N for some central idempotent $e \neq 1$ in Γ. For the case where R = oG is the group ring of a finite group, we shall derive explicit expressions for these annihilators; our expressions include the above-mentioned result of Reiner as a special case.

1. Let C be the maximal order in the center of Γ, and let $\mathfrak{O}$ be a maximal order in Γ that contains R. We define the central conductor to be

$$F(\mathfrak{O}/R) = \{z \mid z\mathfrak{O} \subset R,\ z \in C\}.$$

Since C is contained in every maximal order of Γ, the central conductor is an ideal in C. Now let $\mathfrak{O}$ range over all maximal orders in Γ that contain R, and let F(R) be the C-ideal generated by all the central conductors of R.

THEOREM 1. *For arbitrary* R-*lattices* M *and* N,

$$F(R)\,\mathrm{Ext}_R^1(M, N) = 0.$$

Proof. Let

$$0 \to A \to B \to M \to 0$$

be an exact sequence of R-lattices, where B is projective. Put $kB = k \otimes_o B$, and regard A and B as submodules of kB. Since M is a torsion-free o-module, A is a primitive o-submodule of B; that is, $kA \cap B = A$. Let $\mathfrak{O}$ be a maximal order containing R; then $\mathfrak{O}B$ is the minimal $\mathfrak{O}$-lattice containing B. Now $kA \cap \mathfrak{O}B = \overline{A}$ is an $\mathfrak{O}$-lattice and at the same time a primitive o-submodule of $\mathfrak{O}B$. This implies

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