VALUE DISTRIBUTION AND POWER SERIES WITH MODERATE GAPS

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1. INTRODUCTION

For entire functions it is known (see [1], [3], and [5]) that certain assumptions on the gaps in the power series expansion of the function about zero imply that the function has not only one zero (or a-value) but infinitely many. To obtain corresponding results for functions analytic in the unit disk, it is necessary to link the gap assumption with a growth assumption on the function.

THEOREM 1 (Nevanlinna's notation [4, pp. 4 and 18]). *Let*

$$f(z) = \sum_{k=0}^{\infty} c_k z^{n_k} \tag{1}$$

be analytic in $|z| < 1$, *with* $n_0 = 0$. *Let* $N^0(t)$ *be the number of* n_k *not greater than* t. *If for some fixed* β $(0 < \beta < 1)$

$$N^0(t) = O(t^{1-\beta}) \qquad (t \to \infty), \tag{2}$$

and if

$$n(r, 1/f) = O((1 - r)^{-\lambda}) \qquad (r \to 1),$$

then

$$\log M(r) = O((1 - r)^{-\alpha}) \qquad (r \to 1)$$

for every α *with*

$$\alpha > \max\left(\lambda, \frac{1-\beta}{\beta}\right). \tag{3}$$

COROLLARY. *Let* $f(z)$ *be analytic in* $|z| < 1$ *and of the form* (1), *with* $n_0 = 0$ *and the* n_k *satisfying* (2). *If*

$$\limsup_{r \to 1} \frac{\log \log M(r)}{-\log(1 - r)} \geq \alpha \qquad \left(\alpha > \frac{1-\beta}{\beta}\right),$$

then

$$\limsup_{r \to 1} \frac{\log n(r, 1/f)}{-\log(1 - r)} \geq \alpha.$$

Theorem 1, which has an elementary proof, extends a theorem stated by F. Sunyer I. Balaguer [7]. Related problems in the disk have recently been investigated for larger gaps—Hadamard gaps—by G. and M. Weiss [8] and Ch. Pommerenke [6].

Received June 13, 1966.