LOCAL DEFINITION OF GENERALIZED CONTROL SYSTEMS

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1. INTRODUCTION

An axiomatic way to define general control systems was recently developed by the author [3], [4], [5]. For a more comprehensive bibliographic reference, we refer the reader to [5]; nevertheless, the earlier work of Barbašin [1] should here be mentioned. The main idea consists in the fact that in a control system with some input, the initial conditions $x(t_0) = x_0$ do not determine completely the output $x(t)$, but for each t determine a whole set of attainable points in phase space, called the *attainable set* from (x_0, t_0) at time t, and denoted by $F(x_0, t_0, t)$.

It is possible to characterize satisfactorily such generalized control systems by giving some axioms to be satisfied by the function $F(x_0, t_0, t)$. This was done in the papers mentioned above. Relations with the theory of contingent equations were discussed in [6]. In [7], we gave further applications to stability theory. In all these papers, the general control system was defined over the whole phase space X. Now, especially in stability theory, it is desirable to deal with systems that need to be defined only locally, for example in some neighborhood of a rest point. It is the purpose of this paper to give axioms that define general control systems locally, or more exactly, on a closed subset of the phase space.

2. NOTATION. AXIOMS FOR GENERAL CONTROL SYSTEMS

Let X be a complete, locally compact, metric space (the "state" or "phase" space), and let R be the real line. Points of X will be denoted by small letters, subsets of X by capitals. In order to avoid infinite distances between sets, the given metric $d(x, y)$ may be replaced by

$$\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)}. \tag{2.1}$$

The distance between points and sets and between sets is defined by

$$\rho(a, B) = \rho(B, a) = \inf \{\rho(a, b); b \in B\}, \tag{2.2}$$

$$\beta(A, B) = \sup \{\rho(a, B); a \in A\}, \tag{2.3}$$

$$\alpha(A, B) = \alpha(B, A) = \max \{\beta(A, B), \beta(B, A)\}. \tag{2.4}$$

The ε-neighboring set of a set $A \subset X$ is

$$S_\varepsilon(A) = \{x \in X; \rho(x, A) < \varepsilon\}. \tag{2.5}$$

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