UNILATERAL VARIATIONAL PROBLEMS WITH SEVERAL INEQUALITIES

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1. INTRODUCTION

In a previous paper [6] we have considered nonparametric problems of the calculus of variations in which the "controls" are chosen from a compact Hausdorff space and the admissible curves satisfy given boundary conditions and are restricted to lie in a closed set $A = \{x \in E_n \mid a(x) \leq 0\}$. Here E_n denotes euclidean n-space, and a(x) is a prescribed continuous function on an open subset of E_n with continuous first- and second-order partial derivatives. We shall now extend the results of [6] to the more general problem in which the set A is defined by the simultaneous inequalities $a^k(x) \leq 0$ $(k = 1, \cdots, m)$ and the functions $a^k(x)$ $(k = 1, \cdots, m)$, defined over an open subset of E_n, are twice continuously differentiable.

We shall describe the problem in greater detail and state our assumptions in Section 2. Our basic results are contained in Theorem 3.1, which generalizes Theorem 3.1 of [6] and states that, in a large class of unilateral control problems, there exists a "relaxed" (or generalized) minimizing curve, that this curve can be approximated by solutions of the differential equations of the original problem, and that this relaxed curve satisfies "constructive" necessary conditions for a minimum, including two that are analogous to the Weierstrass E-condition and transversality conditions.

We carry out the proof of Theorem 3.1 in the remaining sections of the paper. Large parts of the proof, especially those contained in Sections 4 and 7, differ only in small details from the arguments of [6].

We refer the reader to [6, Section 1] for a brief discussion of prior work in this general area by Young [8], McShane [3], Filippov [1], Warga [4], and Gamkrelidze [2].

2. STATEMENT OF THE PROBLEM AND ASSUMPTIONS

Let R be a compact Hausdorff space, E_n the euclidean n-space, T the closed interval $[t_0, t_1]$ of the real axis, V an open set in E_n, and B_0 and B_1 closed sets in V. We are also given a function

$$g(x, t, \rho) = (g^1(x, t, \rho), \cdots, g^n(x, t, \rho))$$

from $V \times T \times R$ to E_n and a function $a(x) = (a^1(x), \cdots, a^m(x))$ from V to E_m.

Let $G(x, t) = \{g(x, t, \rho) \mid \rho \in R\}$ $(x \in V,\ t \in T)$, and let F(x, t) be the convex closure of G(x, t).

We define an *original admissible curve with respect to* a(x) as any absolutely continuous function x(t) from T to V such that, for some function $\rho(t)$ from T to R,

(2.1.1) $$dx(t)/dt = \dot{x}(t) = g(x(t), t, \rho(t)) \quad \text{a.e. in } T,$$

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