ON CONSISTENCY OF ℓ-ℓ METHODS OF SUMMATION

H. I. Brown and V. F. Cowling

1. INTRODUCTION

Let ℓ and m denote the space of absolutely convergent series and the space of bounded sequences, respectively. Let A denote an infinite matrix defining a series-to-series transformation that preserves absolute convergence of series, and let ℓ_A denote its absolute summability field. We prove a number of results that relate with one another the concepts of perfectness, reversibility, and type M^*. We also prove some theorems giving conditions for absolute consistency of two matrix methods and for the existence of matrices A with the property that if $f \in \ell'_A$ (the dual space of ℓ_A), then there exists a matrix B such that $\ell_A \subseteq \ell_B$ and $B(x) = f(x)$ for $x \in \ell_A$. Finally, we extend to the class of perfect matrices a theorem that Macphail [2] proved for reversible matrices, and we demonstrate the existence of a nonreversible perfect matrix. Our results belong largely to a class of theorems due to Mazur [3], Mazur and Orlicz [4], Wilansky [6], and Zeller [9].

2. MATRIX MAPPINGS

Let $A = (a_{nk})$ and $x = \{x_k\}$ be a matrix and a sequence of complex numbers, respectively. We write formally

$$y_n \equiv A_n(x) \equiv \sum_k a_{nk} x_k, \tag{1}$$

and we say that the sequence x (and the corresponding series $\sum_k (x_k - x_{k-1})$ with $x_{-1} = 0$) is absolutely summable if each series in (1) converges and $\sum_n |y_n| < \infty$. We say the method is an ℓ-ℓ method provided $\sum_n |y_n| < \infty$ whenever $\sum_n |x_n| < \infty$, and that it is absolutely regular provided in addition $\sum_n y_n = \sum_n x_n$ whenever $\sum_n |x_n| < \infty$. Regarding these concepts the following theorem was proved by Knopp and Lorentz [1] and by Mears [3].

THEOREM (Knopp, Lorentz, Mears). *The matrix* A *defines an* ℓ-ℓ *method if and only if*

$$\sum_n |a_{nk}| \leq M \quad \text{(M independent of k)}. \tag{2}$$

The method A *is absolutely regular if and only if in addition to* (2) *it satisfies the condition*

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