A GAP-THEOREM FOR ENTIRE FUNCTIONS OF INFINITE ORDER

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1. INTRODUCTION AND NOTATION

Let $f(z) = \Sigma a_n z^{\lambda_n}$ be an entire function, and write

$$M(r, f) = \max_{|z|=r} |f(z)|, \qquad m(r, f) = \min_{|z|=r} |f(z)| .$$

In a recent paper, W. H. J. Fuchs [2] proved that if $f(z)$ is of finite order and the sequence $\{\lambda_n\}$ satisfies the "Fabry" gap condition

$$\frac{\lambda_n}{n} \to \infty, \tag{1}$$

then, for each $\varepsilon > 0$, the inequality

$$\log m(r, f) > (1 - \varepsilon) \log M(r, f) \tag{2}$$

holds outside a set of logarithmic density 0.

For functions of infinite order, (1) certainly does not imply (2). In fact, for every sequence $\{\lambda_n\}$ satisfying the condition

$$\sum_1^\infty \frac{1}{\lambda_n} = \infty,$$

A. J. Macintyre [5] has constructed an entire function bounded on the positive real axis. In this paper I shall prove that if the gap condition (1) is replaced by the more stringent condition

$$\lambda_n > n(\log n)^{2+\eta} \tag{3}$$

(for some $\eta > 0$), then (2) holds also for functions of infinite order. It would be desirable to replace condition (3) by the "exact" condition

$$\sum_1^\infty \frac{1}{\lambda_n} < \infty;$$

but this is beyond the scope of our method. The most that could possibly be squeezed out of our method is the replacement of (3) by the condition

$$\lambda_n > n(\log n)(\log \log n)^{2+\eta}.$$

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