BOUNDED CONTINUOUS VECTOR-VALUED FUNCTIONS ON A LOCALLY COMPACT SPACE

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1. INTRODUCTION

In this paper we prove a representation theorem for the strict dual of the space of bounded continuous functions from a locally compact space X into a locally convex linear topological space E. As one would expect, the representation is accomplished by means of an identification between the strict dual and a certain space of measures on X whose values lie in the dual of E. Guided by the now familiar technique of de Branges [3], we apply this representation theorem to obtain a generalized Stone-Weierstrass theorem for the strict topology. The study was inspired by the main results in a paper of R. C. Buck [4], and it may be regarded as an extension of Buck's investigation.

Before stating our main result, we introduce some notations. Let X be a locally compact Hausdorff space, and E a complex linear space with a locally convex topology described by a family of seminorms N. By C(X: E) we denote the space of bounded, complex, continuous functions from X into E, and by $C_0(X: E)$ the subspace of C(X: E) consisting of all functions that vanish at infinity. In case E is the complex field, we denote the spaces by C(X) and $C_0(X)$. The uniform topology σ on C(X: E) is defined by the seminorms

$$\|f\|_p = \sup_{x \in X} p(f(x)),$$

where p ranges over N. A weaker locally convex topology on C(X: E) is the strict topology β defined by the seminorms

$$\|f\|_{\phi,p} = \|\phi f\|_p = \sup_{x \in X} p(\phi(x) f(x)),$$

where ϕ ranges over $C_0(X)$ and p ranges over N. See [4] for general properties, and [4], [7], [9] for applications involving the strict topology.

M(X) denotes the set of all finite, complex, regular Borel measures on X, and $M(X: E^*)$ denotes the set of all measures μ whose values lie in the dual space E^* of E and which satisfy the following conditions:

(1) $\mu(\cdot)s \in M(X)$ for every $s \in E$.

(2) There exist a seminorm $p \in N$ and a constant K such that $\sup |\Sigma \mu(A_i) s_i| \leq K$, where the supremum is taken over all partitions of X into a finite number of disjoint Borel sets $\{A_i\}$ and all finite collections of elements $\{s_i\}$ in E such that $p(s_i) \leq 1$.

THEOREM 1. *If* L *is a strictly continuous linear functional on* C(X: E), *then there exists a* $\mu \in M(X: E^*)$ *such that*

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