ENTIRE FUNCTIONS ON INFINITE VON NEUMANN ALGEBRAS OF TYPE I

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1. INTRODUCTION

The first purpose of this note is to explore the properties of a natural mapping ϕ that is defined on certain homogeneous von Neumann (v.N.) algebras of type I and takes values in a certain Banach space of matrix-valued functions. In the case that the given v.N. algebra is finite, ϕ is a faithful representation, and its properties are well known. In fact, this representation played a central role in the author's study [5] to [8] of finite v.N. algebras of type I. For infinite v.N. algebras of type I, ϕ is a Banach space isomorphism, but it fails to be a representation and has some other unpleasant properties, which we discuss in Section 2.

The second purpose of this note is to use the mapping ϕ to extend a result of A. Brown concerning entire functions on Banach algebras. More precisely, in [1] Brown showed that a necessary and sufficient condition that an entire function f map the algebra $\mathscr{L}(\mathscr{H})$ of all bounded operators on an (infinite-dimensional) Hilbert space $\mathscr{H}$ onto itself is that f map every Banach algebra onto itself. Following Brown, we say that such an entire function has property (U), and we call a Banach algebra $\mathfrak{B}$ adequate if the only entire functions that map $\mathfrak{B}$ onto itself are those with property (U). Several adequate algebras are exhibited in [1], and in Section 3 we set forth a new class of adequate algebras—namely, the infinite v.N. algebras of type I.

2. THE MAPPING ϕ

First we study the above-mentioned mapping ϕ defined on infinite $\aleph_0$-homogeneous v.N. algebras of type I. One knows from [2] or [3] that such a v.N. algebra $\mathfrak{A}$ is unitarily equivalent to a v.N. algebra of the form $\mathfrak{Z} \otimes \mathscr{L}(\mathscr{H}_0)$, in the terminology of [2], where $\mathfrak{Z}$ is an abelian v.N. algebra acting on a Hilbert space $\mathscr{K}$, and where $\mathscr{H}_0$ is a separable Hilbert space. In other words, $\mathfrak{A}$ can be taken to be the algebra of all $\aleph_0 \times \aleph_0$ matrices with entries from $\mathfrak{Z}$ that act as operators on the Hilbert space $\mathscr{K} \oplus \mathscr{K} \oplus \cdots$. A typical element $T \in \mathfrak{A}$ is a matrix (T_{ij}), where the $T_{ij} \in \mathfrak{Z}$. Let $\mathscr{X}$ be the maximal ideal space (or spectrum) of $\mathfrak{Z}$. Then, under the usual topology, $\mathscr{X}$ is an extremely disconnected, compact Hausdorff space, and $\mathfrak{Z}$ is C*-isomorphic to the AW*-algebra $C(\mathscr{X})$ of all continuous complex-valued functions on $\mathscr{X}$. Let $\mathscr{L}$ denote the v.N. algebra of all $\aleph_0 \times \aleph_0$ matrices with scalar entries that act as operators on a separable Hilbert space. Then there is a natural way of associating with each element $T = (T_{ij}) \in \mathfrak{A}$ a function $T(\cdot): \mathscr{X} \to \mathscr{L}$. Namely, let $T(\cdot)$ be the function whose value at $t \in \mathscr{X}$ is the matrix $(T_{ij}(t)) \in \mathscr{L}$, where $T_{ij}(\cdot) \in C(\mathscr{X})$ is the element corresponding to $T_{ij} \in \mathfrak{Z}$. Let $\mathscr{A}$ be the collection of all such functions $T(\cdot)$, and let ϕ denote the mapping $T \to T(\cdot)$. We introduce a metric on $\mathscr{A}$ as follows: Define

$$\|T(\cdot)\| = \sup_{t \in \mathscr{X}} \|T(t)\|,$$

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