NORMAL FUNCTIONS, THE MONTEL PROPERTY, AND INTERPOLATION IN H^∞

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1. INTRODUCTION

The purpose of this paper is to exhibit an interrelationship among the subjects referred to in the title. As a by-product, we obtain new and somewhat shorter proofs of several known results.

2. SOME DEFINITIONS AND KNOWN THEOREMS

Let D denote the open unit disk, let K denote the family of one-to-one conformal mappings of D onto itself, and let f be a function which is meromorphic in D. Then f is said to be *normal* in D if the family $\{f \circ S: S \in K\}$ is normal in D in the sense of Montel. This class of functions was first introduced by Noshiro [9] in 1939; subsequently, it was rediscovered by Lehto and Virtanen [7], who remarked that the sum of a normal function and a bounded function (which is necessarily normal) is normal. Lappan [6] proved

THEOREM 2.1 (Lappan). *Corresponding to each unbounded normal holomorphic function* f *in* D, *there exist a Blaschke product* B *and a normal holomorphic function* g *in* D *such that* fB *and* f + g *are not normal in* D.

Throughout this paper, the Blaschke product

$$\prod_1^\infty \frac{|z_n|}{z_n} \frac{z_n - z}{1 - \bar{z}_n z}$$

is denoted by $B(z; \{z_n\})$. (For a discussion of such products, see [11; p. 274].)

A function defined in D has the *Montel property* if the set of points on the unit circle C where the radial limit exists coincides with the set where the angular limit exists. In [4] the author proved

THEOREM 2.2. *Corresponding to each holomorphic function* f *in* D *having an infinite radial limit at the point* ζ *in* C, *there exists a Blaschke product* B *such that* fB *has an infinite radial limit at* ζ *but fails to have an angular limit there.*

A sequence $\{z_n\}$ of points in D is called an *interpolating sequence* if, given an arbitrary bounded sequence $\{w_n\}$, there exists a bounded holomorphic function f in D such that $f(z_n) = w_n$ $(n = 1, 2, \cdots)$. See [10], where further references are given. Carleson showed that a necessary and sufficient condition for $\{z_n\}$ to be an interpolating sequence is that

Received September 28, 1962.

This work was done under a National Academy of Sciences--National Research Council Postdoctoral Research Associateship at the National Bureau of Standards, 1961-1962, while the author was on leave from Syracuse University.