ON A CERTAIN CLASS OF TRANSFORMATION GROUPS

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1. INTRODUCTION

This note is intended as an appendix to the author's Chapter XV of [1]. The main result [1, Chapter XV, 1.4] of the latter will be referred to in the present note as the CDT (complementary dimension theorem), and it is, in fact, precisely the case $m = 1$ of the main result of this note.

The prototype of the class of transformation groups that we shall study can be described as follows. For each $i = 1, 2, \cdots, m$, let G_i be a closed subgroup of $SO(k_i + 1)$ such that G_i is transitive on the sphere S^{k_i} in the usual action. Let M_i be euclidean space of dimension $k_i + 1$ for $i = 1, 2, \cdots, m$, and let M_0 be euclidean space of some arbitrary dimension. Let $M = M_0 \times M_1 \times \cdots \times M_m$, let n be the dimension of M, and let $G = G_1 \times G_2 \times \cdots \times G_m$. Define an action of G on M as follows. If $g = (g_1, g_2, \cdots, g_m)$, where $g_i \in G_i$, and if $x = (x_0, x_1, \cdots, x_m)$, where $x_i \in M_i$, then let $g(x) = (x_0, g_1(x_1), g_2(x_2), \cdots, g_m(x_m))$. In this note we shall show that, at least as far as cohomology is concerned, the transformation group (G, M) described above is essentially characterized by the fact that

$$\dim F(G_i, M) = n - k_i - 1 .$$

(For the precise statement, see Theorems 1.1 and 4.1 below.)

We shall use the notation of [1]; dim and $\dim_p$ will denote cohomology dimension over Z and over Z_p, respectively (see [1, Chapter I, Section 1.2]). The notation n-cm will be used for n-cm_Z (see [1, I, Section 3]). If X is an n-cm with boundary B, we shall say that a transformation group on X satisfies the hypotheses of the CDT or of Theorem 1.1 if it does so for the naturally related action on X^{dB} (see [1, XV, Section 1.2]). If G acts on a space X and if $Y \subset X$, then we denote by Y^* the image of Y in the orbit space $X^* = X/G$. If K is a subgroup of G, then we denote the identity component of K by K^0, and the normalizer of K in G by $N_G(K)$ (or by N(K) if no confusion can arise).

If G is a compact Lie group acting on a space M, and K is a subgroup of G, we let $M_K = \{y \in M \mid G_y \sim K\}$, which is the set of points with orbits of type (K) (see [1, VIII, Section 2]). If M is an n-cm, we denote a principal isotropy group by H. If furthermore $G = G_1 \times G_2 \times \cdots \times G_m$ and $I \subset \{1, 2, \cdots, m\}$, we let

$$H_I = \prod_{i \in I} (G_i \cap H) \times \prod_{i \notin I} G_i \qquad \text{and} \qquad M_I = \bigcup_{J \subset I} M_{H_J}.$$

We also denote by m(I) the number of elements of $\{1, 2, \cdots, m\} - I$.

If $G_1 \times G_2 \times \cdots \times G_m$ is a compact Lie group acting on the n-cm M, we shall say that condition (A) is satisfied if each of the following three statements is true:

(i) Each G_i is effective on M.

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