ON AN INVARIANT PROPERTY OF SURFACE INTEGRALS

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Our basic tool is the following proposition.

LEMMA. *If* $\alpha = (a_{ij})$ *is an* $n \times n$ *orthogonal matrix, and* $\beta = (b_{\xi\eta})$ *denotes the* $\binom{n}{2} \times \binom{n}{2}$ *matrix whose elements* $b_{\xi\eta}$ *are the determinants of all* 2×2 *submatrices of* α, *then* β *is also an orthogonal matrix.*

We give a proof in Section 2. In Section 3, we use this result to extend the theorems of L. H. Turner [7] concerning the invariance of Cesari's surface integral under orthogonal linear transformations.

1. NOTATION

Let E_n $(n \geq 2)$ be the n-dimensional Euclidean space with an orientation.

If n is a positive integer $(n \geq 2)$, then Ω_n^2 denotes the set of all ordered pairs $\xi = (\xi^1, \xi^2)$ of integers such that $1 \leq \xi^1 < \xi^2 \leq n$. We shall assume that Ω_n^2 is lexiographically ordered.

By the mapping P_n^ξ $(\xi \in \Omega_n^2)$ we mean the projection

$$P_n^\xi(x) = (x^{\xi^1}, x^{\xi^2}) \qquad (x = (x^1, x^2, \cdots, x^n) \in E_n)$$

of E_n onto the hyperplane E_2^ξ.

Let (T, A): $x = T(w)$ $(w \in A)$ be any continuous mapping from an admissible set $A \subset E_2$ into E_n $(n \geq 2)$. Denote by (T^ξ, A) $(\xi \in \Omega_n^2)$ the $\binom{n}{2}$ plane mappings $(P_n^\xi T, A)$ from the admissible set $A \subset E_2$ into $E_2^\xi \subset E_n$. Let $\mathfrak{S}$ be any set of non-overlapping closed simple polygonal regions π in A. If π^* is the oriented boundary of π, then T^ξ maps π^* into an oriented closed curve C_π^ξ in E_2^ξ. For any point $x \in E_2^\xi$, let $O(x; C_\pi^\xi)$ be the topological index of x with respect to C_π^ξ. Then $O(x; C_\pi^\xi)$ is Borel measurable and integrable if (T, A) is cBV. We write

$$u(T^\xi, \pi) = (E_2^\xi)\int O(x; C_\pi^\xi) \quad \text{and} \quad u(T, \pi) = \left[\sum u^2(T^\xi, \pi)\right]^{1/2},$$

where Σ ranges over $\xi \in \Omega_n^2$. (See [1] for the definitions of admissible sets, topological index, and cBV.)

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