FACTORS OF N-SPACE

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1. INTRODUCTION

Bing [2] showed that a certain locally bad 3-gm is a cartesian factor of E^4. Curtis and Wilder [5] showed that the space of Bing, although pathological, is nevertheless locally like E^3 in the sense of homotopy. Raymond [8] proved that every 3-dimensional cartesian factor of E^4 is necessarily locally like E^3 in the sense of homology. Later, Rosen [9] used Bing's construction to show that there exists a nowhere euclidean cartesian factor of E^4. However, it follows easily from our result [7] that his space is a homotopy manifold. It was Curtis [4] who first showed that there exists a cartesian factor of E^4 that is not a homotopy manifold, and who thus answered in the negative a question raised in the original draft of [7]. By constructing a certain pseudo-isotopy of E^{n+1}, by the methods of [2], Andrews and Curtis [1] recently showed that if one shrinks an arc in E^n to a point and then multiplies by the line, then the resulting space is E^{n+1}. In view of [10], this proposition enables us to obtain results similar to those of [9], for all dimensions greater than 2. Furthermore, we can construct the space so that no open subset of it is locally like E^n in the sense of homotopy, and we can replace the construction and argument of [9] by simpler ones. In particular, our construction is similar to one in our earlier work [6]. We also remark that the technique of the present work gives the affirmative answer to a question raised in [6] with the proviso that the construction should be careful.

2. A CERTAIN ARC IN E^n

The following lemma provides us with an arc that we shall use later.

LEMMA 1. *For each* $n \geq 3$, *there exists an arc* P *in* E^n *such that for each open set* U *containing* P *there exists a simple closed curve* C *in* U - P *which is not deformable to a point (that is, whose inclusion map is not null-homotopic) in* E^n - P.

Remark. The arc that we shall use must have a property much stronger than non-simple connectedness of the complement. In the following proof of Lemma 1, we assume the reader's familiarity with the construction of Blankinship [3]. The proof mainly describes what particular set of circles should be avoided in constructing the n-cell E of Blankinship. We use the notation of [3].

Proof of Lemma 1. Let y be the simple closed curve on Bd T that is not deformable to a point in E^n - A. Let y_α be the image of y under the global homeomorphism f_α, where $\alpha = i_1 i_2 \cdots i_j$ ($i_p \leq k$) denotes any array of appropriate positive integers, and $f_\alpha = f_{i_1} f_{i_2} \cdots f_{i_j}$ as in [3]. Let Y be the sum of the sets y_α. We obtain an arc as described in Lemma 1 by avoiding Y in constructing Blankinship's n-cell E and then applying his method.

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