RETRACTS AND EXTENSION SPACES FOR PERFECTLY NORMAL SPACES

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Let Q be a class of topological spaces, and n a nonnegative integer. A topological space Y is called an n-AR(Q) [n-ANR(Q)] if

(a) Y is in Q and

(b) whenever Z is in Q and Y is imbedded as a closed subset of Z with $\dim(Z - Y) \leq n$, then Y is a retract of Z [Y is a retract of some neighborhood of Y in Z].

Y is called an AR(Q) [ANR(Q)] if it satisfies (a) and the statement (b') obtained from (b) by omitting "with $\dim(Z - Y) \leq n$." A space Y is called an n-ES(Q) [n-NES(Q)] if

(a) Y is in Q and

(b) whenever X is in Q, C is a closed subset of X with $\dim(X - C) \leq n$, and $f: C \to Y$ is a continuous mapping, then f has a continuous extension over X [over some neighborhood of C in X] with respect to Y.

Finally, Y is called an ES(Q) [NES(Q)] if Y satisfies (a) and the statement (b') obtained from (b) by omitting "with $\dim(X - C) \leq n$." In the above definitions, dim X means the dimension of X defined in terms of finite open coverings.

A normal space X is called *perfectly normal* if every closed subset of X is a G_δ. Every metric space is perfectly normal, and every perfectly normal space is countably paracompact [1, p. 221]. Some justification for our interest in the class of perfectly normal spaces is provided by the following theorem of M. Katětov [7].

THEOREM. *Let* B *be a separable Banach space,* K *a convex subset of* B, *and* C *a closed set of type* G_δ *in a normal space* X. *Then every continuous mapping* $f: C \to K$ *has a continuous extension* $F: X \to K$ *with*

$$\dim F(X - C) \leq \min[\dim C + 1, \dim f(C) + 1, \dim X].$$

The object of this paper is to prove the following five theorems.

THEOREM 1. *Let* Y *be a separable metric space. Then the following implications hold between the statements listed below:* (a) *is equivalent to* (d) *and* (b) *is equivalent to* (c); *moreover,* (b) *implies* (a) *and* (c) *implies* (d).

(a) Y *is* LC^{n-1}.

(b) Y *is an* n-ANR (perfectly normal).

(c) Y *is an* n-NES (perfectly normal).

(d) *If* X *is perfectly normal,* $\dim X \leq n$, *and* C *is closed in* X, *then any continuous* $f: C \to Y$ *has a continuous extension over some neighborhood of* C *in* X *with respect to* Y.

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