SOME CHARACTERIZATIONS OF HOMOLOGICAL DIMENSION

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1. Let X be a compact Hausdorff space, and let G be an abelian group. The *homological dimension of* X *relative to* G is the largest integer n such that there exists a pair (A, B) of closed subsets of X whose n-dimensional Čech homology group $H_n(A, B: G)$ is not zero. By $D_*(X: G)$ we shall denote the homological dimension of X. We have the relation $\dim X \geq D_*(X: G)$, for each group G. The equality $\dim X = D_*(X: G)$ does not necessarily hold. For example, for any positive integer n, there exists a continuum X such that $\dim X = 2n$ and $D_*(X: G) = n$ for each finitely generated abelian group G.

Let N be a class of compact Hausdorff spaces. A countable system

$$\{T_i(G);\ i = 1, 2, \cdots\}$$

of locally compact fully normal spaces is called a *T-system for the group* G *with respect to the class* N if, for each X of N, we have the equality

$$D_*(X: G) = \mathrm{Min}\,\{\dim (X \times T_i(G)) - \dim T_i(G);\ i = 1, 2, \cdots\}\ .$$

If a T-system for G with respect to N consists of only one space, then the space is called a *test space for* G *with respect to* N (see [7]). The following notations will be used throughout this paper.

Z: the additive group of all integers.

Z_q: the cyclic group Z/qZ of order q.

R: the additive group of all rational numbers.

R_1: the additive group of all rational numbers reduced mod 1.

Q_p: the p-primary component of R_1.

$Z(\alpha_p)$: the limit group of the inverse system

$$\{Z_{p^i},\ i = 1, 2, \cdots;\ h_i^{i+1}: Z_{p^{i+1}} \to Z_{p^i}\}\ ,$$

where the homomorphism $Z_{p^{i+1}} \to Z_{p^i}$ is a natural homomorphism induced by the inclusion $p^{i+1} Z \subset p^i Z$.

L: the class of all finite-dimensional compact Hausdorff spaces.

L_n: the class of all n-dimensional compact Hausdorff spaces.

$L_n(G)$: the class of all finite-dimensional compact Hausdorff spaces X such that $\dim X - D_*(X: G) = n$.

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