ISOTROPY STRUCTURE OF COMPACT LIE GROUPS ON COMPLEXES

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1. INTRODUCTION

In this paper we prove the following conjecture of Floyd [1, page 95]:

THEOREM. *If* G *is a compact Lie group operating on a finite complex* K, *then there are only finitely many distinct conjugate classes of isotropy subgroups.* In the proof we use a decomposition of K into a finite number of invariant open manifolds, each of which has an orientable covering manifold whose integral cohomology (with compact supports) is finitely generated. Lifting the action of G to the covering manifolds, we apply a result of Mann [4] to establish the theorem.

The theorem is false when K is a locally-finite complex having finitely generated integral cohomology. To see this, consider the 2-complex consisting of a line and a sequence of closed discs, with centers on the line and going off to infinity. Define an action of the circle group S^1 on this complex by defining θ in S^1 $(0 < \theta < 2\pi)$ to act as the rotation $j\theta$ on the j^{th} disc. Then Z_j (the subgroup of S^1 isomorphic to the integers, modulo j) leaves the j-th disc point-wise fixed, and therefore Z_j is an isotropy subgroup for each j. On the other hand, the one-point compactification of this complex is of the same homotopy type as the circle, and therefore the finitely generated integral cohomology condition is satisfied.

2. CONSTRUCTION OF COVERING MANIFOLDS WITH FINITELY GENERATED COHOMOLOGY

Let K be any n-dimensional complex. Denote by F(K) the subset of K consisting of points which have neighborhoods homeomorphic to E^n. Then F(K) is an n-manifold. If F(K) is connected and K = Cl [F(K)], then K will be said to be *F-connected.*

LEMMA 1. *Let* K *be a finite* n*-complex which is* F*-connected and such that* F(K) *is a non-orientable* n*-manifold. Then there exist a finite* n*-complex* K* *and a simplicial map* p: K* → K *such that*

$$p \mid p^{-1}(F(K)) \colon p^{-1}(F(K)) \to F(K)$$

is the orientable double covering of F(K) *and* $p^{-1}(F(K))$ *has finitely generated cohomology.*

Proof. Let K^{n-1} be the (n - 1)-skeleton of K, let A be the subcomplex formed from the union of all (n - 1)-simplexes which are faces of exactly two n-simplexes, and let B be the union of all the other (n - 1)-simplexes in K^{n-1}. Note that

$$F(K) \subset K - B \subset (K - K^{n-1}) \cup A.$$

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