FINITE ORBIT STRUCTURE ON LOCALLY COMPACT MANIFOLDS

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1. INTRODUCTION

The following conjecture was raised by D. Montgomery. *If a compact Lie transformation group* G *operates on a compact manifold* X, *then there are only a finite number of distinct conjugate classes of isotropy subgroups* G_x $(x \in X)$. The conjecture was answered in the affirmative through the efforts of Floyd [3] and Mostow [6]. It is also known that if the manifold X is only locally compact, then *locally* there are but a finite number of classes of isotropy subgroups [1, VII]. However, in the case where the manifold X is not compact, a counterexample due to Montgomery exists for the conjecture. In this counterexample [8], a circle group operates on a 3-manifold having infinitely generated integral cohomology, and the action produces an infinite number of distinct isotropy subgroups. It is precisely this counterexample which suggests the main result of this present paper: the conjecture still holds when X is an orientable cohomology manifold over the integers with finitely generated integral cohomology (Theorem 3.5). Alexander-Spanier cohomology with compact supports is used in the above.

Our techniques are based upon those of Floyd's paper [3], in that we establish a main lemma of the form (3.3) in Section 3. The author is grateful to Professor Floyd for bringing this problem to his attention. The results of this paper will be used, in a later paper, to investigate the action of compact Lie groups on complexes. The question of whether the orientability of X may be dropped in (3.5) is still open.

2. DEFINITIONS AND PRELIMINARY RESULTS

X will always be considered as a locally compact Hausdorff space, and L as a principal ideal domain. $H_c^i(X; L)$ will denote the i-th Alexander-Spanier cohomology group of X with compact supports. In all applications, L will be either Z, the ring of integers, or Z_p, the prime field of characteristic p. $H_c^*(X; L)$ will denote the direct sum of the groups $H_c^i(X; L)$, and reduced cohomology will be used for the 0-dimensional groups.

For an open subset U of X and a closed subset A of X, we have the standard homomorphisms

$$j_{XU}^*: H_c^*(U; L) \to H_c^*(X; L) \quad \text{and} \quad r_{XA}^*: H_c^*(X; L) \to H_c^*(A; L).$$

In fact, we have the following exact cohomology sequence for U open in X:

$$(2.1) \qquad \to H_c^i(U; L) \xrightarrow{j_{XU}^i} H_c^i(X; L) \xrightarrow{r_{X,X-U}^i} H_c^i(X - U; L) \to H_c^{i+1}(U; L) \to \cdots.$$

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