ASYMPTOTIC EQUIVALENCE AND ASYMPTOTIC BEHAVIOUR OF LINEAR SYSTEMS

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1. Two systems of differential equations are said to be asymptotically equivalent if, corresponding to each solution of one system, there exists a solution of the other system such that the difference between these two solutions tends to zero. If we know that two systems are asymptotically equivalent, and if we also know the asymptotic behaviour of the solutions of one of the systems, then it is clear that we can obtain information about the asymptotic behaviour of the solutions of the other system. In some cases, we can reduce the problem of asymptotic equivalence to the problem of proving that a certain system has solutions which tend to any prescribed limit. Then, by using slight extensions of some results of Wintner [10], we obtain alternate proofs of some theorems of Wintner [8] on asymptotic equivalence, as well as some results analogous to those of Levinson [6] and Yakubovič [12] on asymptotic behaviour of linear perturbations of linear systems with constant coefficients. In general, our hypotheses are considerably more stringent than Levinson's, but our error estimates are correspondingly sharper. The same approach yields results analogous to those of Cesari [2] and Levinson [6] on asymptotic behaviour of certain linear systems. Here too, our hypotheses are more stringent, but our error estimates are sharper than earlier results. Unfortunately, we see no way to obtain the earlier results from ours. Our proofs are simpler than previous proofs, in that they avoid conversion to integral equations and construction of successive approximations. Instead, they depend on the results of Wintner [10] mentioned earlier, whose proofs are quite elementary.

We shall always write systems of differential equations in the usual vector form (see for example [3] or [4]). The norm $|x|$ of a vector x will always mean the sum of the absolute values of the components of x, and the norm $|A|$ of a matrix A will always mean the sum of the absolute values of the elements of A. All derivatives with respect to t will be denoted by $'$, and the derivative of a vector or matrix will always mean the vector or matrix respectively whose elements are the derivatives of the elements of the original vector or matrix. We shall assume that all coefficients are continuous, without always stating this explicitly.

2. We wish to compare the solutions of the linear system

$$x' = A(t)x \tag{1}$$

with the solutions of a perturbed system

$$y' = A(t)y + f(t, y). \tag{2}$$

Sometimes we shall be interested in a linear perturbation giving a system

$$y' = [A(t) + B(t)]y. \tag{3}$$

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