

PARTNERSHIP GAMES WITH SECRET SIGNALS PROHIBITED

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The ethics of bridge prohibit the use of secret signals by either partnership. This is explicitly stated in Law 5 of *The Laws of Duplicate Contract Bridge* (see [2], pp. 223-224). Two game-theoretical formalizations of this rule are:

1. *Whenever an agent of either player is required to make a bid or to play a card as defender, he must announce his behavioral strategy as well as the bid or play which results from the randomization required by that behavioral strategy.*
2. *Instead of announcing the behavioral strategy to all other agents, the agent who is moving announces it to a referee. The referee announces to the other agents their a posteriori probabilities for the hands unseen by them under the assumption of the previous sequence of bids and plays.*

These formalizations are never equivalent, as considerably more information is divulged in the first than in the second. In Section 4, we give a sufficient condition that, from the standpoint of the value of the game, the information given in Formalization 2 is as good as that in Formalization 1.

In Section 1, an abstract version of bridge will be introduced.

In Section 2 it will be shown that abstract bridge with all agents always required to announce behavioral strategies is a perfect information game according to the definition by Blackwell and Girshick [1, Definition 1.7.1]. Hence, with this rule the game has a pure value.

In actual bridge, a declarer is not required to announce his behavioral strategies. Thus, in abstract bridge we may wish to allow an agent to stop announcing his behavioral strategies at some point in the game when he knows that his partners are not required to make any further choices. An example due to Fox [3] shows that in this case the game may not have a value.

In Section 3 it is shown that the game does have a value provided that, if an agent is no longer required to announce behavioral strategies, then his actions at each of his moves are restricted to a finite set.

In Section 4 we again assume that all behavioral strategies are announced. It is shown that if each agent has perfect information about the other agents' actions, then ϵ -good strategies exist in which the behavioral strategies depend only on the a posteriori probabilities for the deal.

In Section 5 it is shown that both players have good strategies provided the action-spaces are all finite. This is true even if behavioral strategies are not always announced.

Thompson [7] has given an example of a simplified bridge game in which the optimal strategies differ according to whether secret signals are or are not permitted. His example is valid even for our more general strategy space, in the case of no secret signals.