A SIMPLIFIED PROOF OF WARING'S CONJECTURE

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The purpose of this paper is to give a short-cut in the proof of Waring's conjecture. The novelty in our procedure lies in the use of some of the elementary number theoretic notions due to Schnirelmann, which allow us to employ crude upper bounds in the circle method, rather than the usual asymptotic formulae.

Our starting point is the estimate for the "Weyl sums" (see [3, Volume I, Part 2, p. 255]), a special form of which we state as our

LEMMA 1. *Let* $k > 1$ *be a fixed integer. There exists a* $\delta > 0$ *and a* C_1 *such that, for any positive integers* N, a, b *with* $(a, b) = 1$ *and* $N^{1/2} \leq b \leq N^{k-1/2}$,

$$\left| \sum_{n=1}^{N} e\left(\frac{a}{b} n^k\right) \right| \leq C_1 N^{1-\delta}.$$

(Throughout, we write $e(t) = e^{2\pi i t}$, and C_1, C_2, $\cdots$ denote constants.) Our end-point will be the

THEOREM. *If, for each positive integer* s, *we write*

$$r_s(n) = \sum_{\substack{n_1^k + \cdots n_s^k = n \\ n_i \geq 0}} 1,$$

then there exist g *and* C *such that* $r_g(n) \leq C n^{g/k-1}$ *for all* $n > 0$.

The previously cited notions of Schnirelmann allow the deduction, from this theorem, of the full Waring result, namely:

There exists a G *for which* $r_G(n) > 0$ *for all* $n > 0$. For the details see [1, pp. 40, 41].

To prove our theorem: since

$$r_g(n) = \int_0^1 \left(\sum_{m \leq n^{1/k}} e(xm^k) \right)^g e(-nx)\, dx,$$

it suffices to prove that there exist g and C for which

$$\int_0^1 \left| \sum_{n=1}^{N} e(xn^k) \right|^g dx \leq C N^{g-k} \quad \text{for all } N > 0. \tag{1}$$

First some parenthetical remarks about this inequality: Suppose it is known to hold for some C_0 and g_0; then, since $\left| \sum_{n=1}^{N} e(xn^k) \right| \leq N$, it persists for C_0 and any

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