AUTOMORPHIC FORMS OF NONNEGATIVE DIMENSION AND EXPONENTIAL SUMS

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I. INTRODUCTION

In this paper we extend the methods and results of the previous paper [2]. There we discuss the three groups $G(\sqrt{l})$ $(l = 1, 2, 3)$ of linear fractional transformations of the upper half-plane $\Im\tau > 0$ onto itself, where $G(\sqrt{l})$ is generated by the two transformations $S(\tau) = \tau + \sqrt{l}$ and $T(\tau) = -1/\tau$, and we construct automorphic forms of nonnegative *even* integral dimension, with multiplier system identically one, for these groups. Of course, $G(1)$ is the modular group.

Here the results of [2] are extended in the following way. The same groups are considered, but now we construct forms of *arbitrary* integral dimension $r \geq 0$, with arbitrary multiplier systems. Specifically, let Γ denote any one of the three groups in question and let $M \in \Gamma$, $M\tau = (\alpha\tau + \beta)/(\gamma\tau + \delta)$. Given any integer $r \geq 0$, we construct functions $F(\tau)$ that are regular in $\Im\tau > 0$ and satisfy throughout this half-plane and for all $M \in \Gamma$ the condition

$$(1.1)\qquad F(M\tau) = \varepsilon(M)\cdot(-i(\gamma\tau + \delta))^{-r}\cdot F(\tau),$$

where $\varepsilon(M)$ does not depend on τ and $|\varepsilon(M)| = 1$ for all $M \in \Gamma$.

With each transformation $M \in \Gamma$ we associate the two matrices

$$M = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \quad\text{and}\quad -M = \begin{pmatrix} -\alpha & -\beta \\ -\gamma & -\delta \end{pmatrix};$$

in this context we shall not distinguish between the two matrices. Therefore, applying (1.1) with M replaced by $-M$, we see that

$$(1.2)\qquad \varepsilon(-M)(-i(-\gamma\tau - \delta))^{-r} = \varepsilon(M)(-i(\gamma\tau + \delta))^{r}.$$

Now, when there exists a function $F(\tau)$ satisfying (1.1) it follows in a simple fashion that if

$$M_1 = \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \in \Gamma, \qquad M_2 = \begin{pmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{pmatrix} \in \Gamma,$$

then

$$(1.3)\qquad \varepsilon(M_1 M_2)(-i(\gamma_3\tau + \delta_3))^{-r} = \varepsilon(M_1)\varepsilon(M_2)(-i(\gamma_1 M_2\tau + \delta_1))^{-r}(-i(\gamma_2\tau + \delta_2))^{-r},$$

where $M_1 M_2 = \begin{pmatrix} * & * \\ \gamma_3 & \delta_3 \end{pmatrix}$. The multipliers $\varepsilon(M)$ are said to form a *multiplier system for* Γ *corresponding to the dimension* r, provided $\varepsilon(M)$ is a complex-valued function

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