p-ADIC TRANSFORMATION GROUPS

Chung-Tao Yang

1. INTRODUCTION

The present paper is motivated by considerations of the question whether a p-adic group can act effectively as a topological transformation group on a manifold. Our purpose is to study the topological transformation groups (G, X) in which G is a p-adic group and X is a locally compact Hausdorff space. We prove that if X is of homology dimension not greater than n (with respect to reals modulo 1), the homology dimension of the orbit space X/G is not greater than n + 3. If in particular X is an n-dimensional manifold and G acts effectively on X, then the homology dimension of X/G is actually equal to n + 2.

From our result it is easy to verify the following known theorem. If G is a p-adic group (respectively, a p-adic solenoid group) acting *freely* on an n-dimensional manifold X, then the orbit space X/G is of dimension either n + 2 (respectively, n + 1) or ∞. It remains to be seen whether our results can be used to prove the well-known conjecture that a p-adic group can not act effectively on a manifold.

In proving our results, we make extensive use of a modified special homology theory of Smith in which reals modulo 1 are used as coefficients. For any compact Hausdorff space X on which a prime-power order cyclic group or a p-adic group acts, special homology groups are defined and several exact sequences are established.

2. COVERINGS

Let X be a space, and A a subset of X. An *open covering* of A in X is a collection λ of open subsets of X such that every $U \in \lambda$ intersects A and such that the union $\bigcup\{U \mid U \in \lambda\}$ contains A. A *closed covering* of A in X is a collection of closed subsets of X the interiors of which form an open covering of A in X. By a *covering* we mean either an open covering or a closed covering.

Let λ and μ be coverings of A in X. If every member of μ is contained in some member of λ, we say that μ *refines* λ. For every $V \in \mu$, the *star* of V in μ, denoted by (V, μ), is defined to be the union of the members of μ which meet V. If for every $V \in \mu$, (V, μ) is contained in some member of λ, we say that μ *star-refines* λ.

(2.1) Let X be a compact Hausdorff space, and A a closed subset of X. Then, for every open covering λ of A in X, there exists an open covering μ of A in X star-refining λ.

(2.2) Let X be a normal space, and A a closed subset of X. Let α be an open covering of A in X. Then for every finite closed covering $\lambda = \{F_1, \cdots, F_m\}$ of A in X refining α there exists an open covering $\mu = \{U_1, \cdots, U_m\}$ of A in X refining α such that every F_i is contained in U_i and such that $F_{i_1} \cap \cdots \cap F_{i_j} \neq \emptyset$ if and only if $U_{i_1} \cap \cdots \cap U_{i_j} \neq \emptyset$.

Received February 18, 1960
This paper was prepared while the author was under ONR contract.