THE FOURIER COEFFICIENTS OF AUTOMORPHIC FORMS ON HOROCYCLIC GROUPS, III

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1. INTRODUCTION

In previous papers, we have shown how the circle method can be used to determine the Fourier coefficients of *entire* automorphic forms on certain horocyclic groups *(Grenzkreisgruppen)*. In the present paper, we extend the results to automorphic forms that have poles.

Our results are contained in the following theorems. For definitions of symbols, see [2], except as noted; automorphic forms with poles are defined in Section 2 of the present paper.

THEOREM 1. *Let* $F(z)$ *be an automorphic form of dimension* $r > 0$ *on an* H-*group* Γ. *Let* R_0 *be a fixed fundamental region of* Γ *which does not have* ∞ *as a vertex, and let* $p_1, p_2, \cdots, p_s$ *be a complete set of inequivalent vertices of* Γ. *Let the expansions of* $F(z)$ *at the parabolic points be*

$$(1.1)\qquad \begin{aligned} F(z) &= (A_k z)^{-r} t_k^{\alpha_k} f_k(t_k) \qquad (t_k = e(A_k z/\lambda_k)),\\ f_k(t) &= \sum_{m=-\mu_k}^{\infty} a_m^{(k)} t^m \qquad (1 \leq k \leq s). \end{aligned}$$

Let $F(z)$ *have simple poles at the interior points* $z_1, z_2, \cdots, z_q$ *of* R_0 *with residues* $B_1, \cdots, B_q$. *Then, for each* k $(1 \leq k \leq s)$, *the Fourier coefficients* $a_m^{(k)}$ *with* $m \geq 0$ *are given in terms of the set of coefficients* $a_m^{(j)}$ *with* $m < 0$ $(1 \leq j \leq s)$ *by the formula*

$$(1.2)\qquad \begin{aligned} a_m^{(k)} = \left(\frac{2\pi}{\lambda_k}\right) e(r/4) \sum_{j=1}^{s} \sum_{\nu=1}^{\mu_j} a_{-\nu}^{(j)} \sum_{c_{jk} \in C'_{jk}} c_{jk}^{-1} A(c_{jk}, \nu_j, m_k)\, L(c_{jk}, m_k, \nu_j, r)\\ - \left(\frac{2\pi i}{\lambda_k}\right) \sum_{n=1}^{q} B_n \sum_{V \in \Delta_k(z_n)} \varepsilon(V)\, e(-(m+\alpha_k) A_k V z_n/\lambda_k)\, (A_k V z_n)^{r+2} (c z_n + d)^{-r-2}. \end{aligned}$$

Here C'_{jk} *is the set of positive elements of* C_{jk}, *and* Δ_k *is defined in (4.5).*

Poles of higher order can be treated in an analogous manner, but the algebraic details, into which we do not enter here, become rather complicated.

THEOREM 2. *If* $F(z)$ *is an automorphic form of dimension zero on* Γ *having the expansions (1.1), then, for* $m \geq 1$, $a_m^{(k)}$ *is given by the series (1.2) with an error term* $O(1)$ $(m \to \infty)$, *where in the first series* c_{jk} *is further restricted by the*

Received June 16, 1959.

Work performed under the auspices of the U. S. Atomic Energy Commission. Presented to the American Mathematical Society, January 22, 1959.