ORIENTATION IN GENERALIZED MANIFOLDS AND APPLICATIONS TO THE THEORY OF TRANSFORMATION GROUPS

Glen E. Bredon

0. INTRODUCTION

This paper is concerned with a type of cohomology manifold which is of importance in the theory of topological transformation groups. These spaces have been studied by C. T. Yang, D. Montgomery, P. A. Smith, E. E. Floyd, A. Borel, and others.

The paper actually consists of two rather disjointed parts. The first consists of Sections 2 through 6 and deals with orientable coverings, and the second consists of Sections 7 through 11 and deals with the theory of transformations of prime period. These parts have a few points of contact in Section 7, and both use the preliminary material of Section 1, which contains some general facts about generalized manifolds.

In Section 2 we define the orientable covering of a cohomology manifold. The definition follows classical lines closely, except that the orientable covering has, in general, more than two sheets. The special case in which the group of coefficients is the integers is studied in Section 4, along with the relationship of this case to the general case. In this case, as in the classical case, the orientable covering has two sheets.

In Section 5 we apply our methods to study the lifting of transformation groups to the orientable covering, and we find that a group may be lifted in a unique manner to be a group of orientation-preserving transformations on the orientable covering. In Section 6 we study conditions under which the lifted group is a topological transformation group of the orientable covering if it is a topological transformation group of the original space.

In Section 7 we study groups of transformations of prime order p on a cohomology manifold over Z_p, that is, P. A. Smith's theory. We obtain, in a modern form, Smith's theorems that the fixed point set is a cohomology manifold over Z_p and is orientable if the space M is orientable; we also obtain a partial converse: If M is paracompact, and if the fixed point set is orientable, then it has an orientable neighborhood in M. We also obtain a new dimensional parity theorem which asserts that if the prime p is odd, then the dimensions of M and of each nonempty component of the fixed point set are of the same parity. Analogous dimensional parity theorems for spaces possessing the homology groups of a sphere have been given by P. A. Smith [10], E. E. Floyd [4], and A. Borel [2]. (See also Section 11 of the present paper.) The analogue of the refinement of the theorem necessary for the case p = 2, given by Liao [7], is also proved, and in the course of its proof we find the local groups in the orbit space about a fixed point. The results of this section are mainly obtained by studying the relationships between the orientations "in the small" of the fixed point set and of M.

In Section 9 we give another proof of the dimensional parity theorem of Section 7, under slightly more restrictive assumptions; it is based on entirely different techniques and has the advantage of being somewhat shorter than the preceding proof.

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