SEPARATION AND UNION THEOREMS FOR GENERALIZED MANIFOLDS WITH BOUNDARY

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INTRODUCTION

Generalized manifolds are a class of spaces which reflect many of the local and global homology properties of locally Euclidean manifolds. Moreover, they form a class of spaces in which certain operations are closed with respect to this class, whereas this may not be true for classical manifolds; for example, if a generalized manifold is the Cartesian product of two spaces, then both factors are generalized manifolds. (See Theorem 6 for a proof of this fact.)

It is well-known that every 2-sphere imbedded in a 3-sphere separates the 3-sphere into two open connected sets both of whose frontiers coincide with the 2-sphere. However, neither open set need be an open 3-cell, and even if the sets are 3-cells, the 2-sphere may not fit onto a complementary domain to form a closed 3-cell. Wilder [10] has shown that these complementary domains, nevertheless, are generalized cells. This follows as an application of the Jordan-Brouwer separation theorem and its converse [10; Chap. 10]. Wilder's theorems are cast in the framework of orientable, sphere-like, compact generalized n-manifolds over a field. P. A. White [9], has extended the results by relaxing some of the sphere-like conditions. The purpose of this paper is to extend the results to noncompact, locally orientable generalized n-manifolds with boundary defined over an arbitrary principal ideal domain (see Section 3).

The separation of a generalized n-manifold with boundary by a generalized (n - 1)-manifold with boundary is converse to the problem of showing that the union, along the boundary, of two generalized n-manifolds with boundary is again a generalized n-manifold with boundary. Consequently, we shall treat both problems.

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1. NOTATION AND DEFINITIONS

By a *space* we shall mean a locally compact Hausdorff space. By a *neighborhood of a point* in a space we shall mean any open set containing the particular point. If A is a subset of the space X, then by A^- we shall mean the closure of the subset A in X.

The p*-dimensional Čech cohomology group of* X *with compact supports and coefficients in the principal ideal domain* L will be denoted by $H_c^p(X; L)$. However, in general, we shall omit the ring L from the notation, since no confusion can arise. If U is an open subset of the space X, then the sequence

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