A NOTE ON THE LOCAL "C" GROUPS OF GRIFFITHS

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In [1], Griffiths defined a local group $C_r(x; G)$ which extends the notion of non-local r-cut point (see [6]; Chap. 7). This group was used by Griffiths to obtain a local theorem of Hurewicz type. In addition, homology manifolds over the integers were defined, and a duality which amounts to Poincaré duality with respect to the rationals was proved.

The result of this note was used by the author, in a paper presented to the American Mathematical Society, to answer Griffiths' question whether his duality theorem can be extended to the torsion coefficients provided that the homology manifold, in his new sense, is LC^n. The result was also used to show that an LC_0^n separable metric space is a (locally orientable) generalized manifold in the sense of Wilder if and only if it is a (locally orientable) singular homology manifold (see [4] for the definition of singular homology manifold). Subsequently, S. Mardešić has informed me that he has shown, by a modification of [3], that for lc_0^n (in the singular sense), locally compact, Hausdorff pairs there exists an isomorphism, up through dimension n, of the singular homology groups onto the Čech homology groups (with compact carriers and an arbitrary group as coefficients). Griffiths has proved the same theorem in [2]. Thus one can now successfully by-pass the local groups of Griffiths to obtain Poincaré duality, using arbitrary coefficients and Čech homology, provided that the space is lc_0^∞ in the singular sense as follows:

A locally compact, finite-dimensional, lc_0^∞ *(in the singular sense) space* X *is a locally orientable, singular homology* n*-manifold with respect to an arbitrary coefficient group* G *if and only if*

(a) *for each* $x \in X$, $\check{H}_p(X, X - x) \approx \begin{cases} 0 & (p \neq n), \\ G & (p = n); \end{cases}$

(b) *for each* $x \in X$, *there exists an open* U_x *such that*

$$i_*: \check{H}_n(X, X - \bar{U}) \to \check{H}_n(X, X - y)$$

is an isomorphism onto for all $y \in U$.

($\check{H}$ denotes the Čech homology with compact carriers.)

Hence, the duality theorems [4] for lc_0^∞, locally orientable, singular homology n-manifolds can be stated either in terms of the Čech homology with compact carriers or in terms of the singular homology.

As an interesting consequence it follows that *an* lc_0^∞ *(in the singular sense), locally orientable generalized manifold in the sense of Wilder is locally separable metric and, furthermore, that it is metrizable if and only if it is paracompact.* This is a direct consequence of [4, 3.9, Remark 1].

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