A GENERALIZATION OF THE LOTOTSKY METHOD OF SUMMABILITY

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1. INTRODUCTION AND NOTATION

In a recent paper [5], A. V. Lototsky introduced a new method of summability which possesses interesting and important properties. Later, in [2], R. P. Agnew gave simplified proofs of Lototsky's results as well as some further properties of the new method. In this paper we shall show that many results proved for the Lototsky (or L) method of summability are valid for a general class of transformations to which the L transformation belongs.

Corresponding to a sequence $\{d_n\}$ with $n \geq 1$ (for all sequences in this paper, the index denoting the order of the terms assumes the values 0, 1, 2, ⋯, except where it is stated otherwise), the symbol $d_n!$ will denote the product $d_1 d_2 \cdots d_n$, and *not*, as is sometimes the case, the function $\Gamma(d_n + 1)$. Given a sequence $\{d_n\}$ $(n \geq 1)$, we shall denote by $\{p_{nm}\}$ $(m = 0, \pm 1, \pm 2, \cdots;\ n = 0, 1, \cdots)$ the double sequence defined by

$$(1.1)\qquad \begin{cases} p_{00} = 1, \\ (x + d_n)! \equiv \prod_{m=1}^{n} (x + d_m) \equiv \sum_{m=0}^{n} p_{nm} x^m, \\ p_{nm} = 0 \quad \text{for } m > n \text{ and } m < 0. \end{cases}$$

It is easy to see that

$$(1.2)\qquad p_{n+1,m} - d_{n+1} p_{nm} = p_{n,m-1} \quad \text{for } m = 0, \pm 1, \cdots;\ n = 0, 1, \cdots.$$

2. THE $[F, d_n]$ TRANSFORMATIONS

Suppose a fixed sequence $\{d_n\}$ $(n \geq 1)$ satisfying $d_n \neq -1$ for $n \geq 1$ is given. Then $\{t_n\}$, the $[F, d_n]$ transform of $\{s_n\}$, is defined by

$$t_0 = s_0.$$

$$t_n = [(1 + d_n)!]^{-1} \sum_{m=0}^{n} p_{nm} s_m \qquad (n \geq 1).$$

The L transformation was defined as a transformation which, applied to a sequence $\{s_n\}$ defined for $n \geq 1$, yields a transform $\{t_n\}$ defined for $n \geq 1$, also. Now for

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